A Note on Candidate Stability for Separable Preferences[*]

Kentaro Hatsumi[†]

First Version: January 22, 2009
This Version: April 11, 2011

Abstract: We consider the election model to select a subset from a set of candidates with separable preferences, and axiomatically investigate a rule satisfying candidate stability, which is the requirement to deter any candidate from strategic withdrawal. If a candidate merely wishes to be elected, candidate stability is equivalent to the requirement that the outcome when a candidate is not running equals the outcome of dropping herself. It is also equivalent to the independence of the selection for each candidate. We also consider two other restrictions to candidates' preferences, and derive an equivalent condition to candidate stability in each case.

Keywords: social choice, mechanism design, multi-winner election, strategic candidacy

JEL Classification Numbers: D71, D72

[*]I am grateful to Kazuhiko Hashimoto, Yusuke Samejima, Shigehiro Serizawa, William Thomson, and other participants at the 2010 Japanese Economic Association Spring Meeting and seminars at the Universities of Osaka and Rochester for their helpful comments. I acknowledge the financial support from the Japan Society for the Promotion of Science via the Research Fellowship for Young Scientist.

[†]Institute of Economic Research, Kyoto University. Yoshida, Sakyoku, Kyoto 606-8501, Japan. E-Mail: kentaro.hatsumi *at* fx5.ecs.kyoto-u.ac.jp

1 Introduction

We axiomatically investigate a rule in the multi-winner election model where voters can select any subset from a set of candidates. Examples are where the members of a club select newcomers or where the executive committee of a society selects new members. Barberà et al. (1991) first study this model.[1] We extend their model so that not only voters but also candidates have preferences over the power set of the sets of candidates, and the actual running candidates is variable. In this model, we investigate a rule satisfying "candidate stability", which requires that for any candidate, standing in the election is at least as desirable as withdrawing from it. Dutta et al. (2001) first axiomatically investigate *candidate stability.* They support this axiom as follows. Consider the two-stage game where in the first stage candidates decide whether to run for election, and in the second stage, voters choose the winner(s) from the set of actually running candidates. If the election rule in the second stage satisfies *candidate stability*, then the decision to run by any candidate constitutes a Nash equilibrium in the first stage. Then we need not seriously consider strategic behavior by candidates in the first stage.[2]

Throughout the paper, we assume that the sets of voters and candidates are disjoint. In reality, voters and candidates are separated in many elections, and even though they may overlap, our assumption well approximates elections where the number of candidates is relatively small compared with the number of voters. We assume that candidates possess "separable" preferences, which requires that for any voter, a candidate x is preferred to the null outcome if and only if any set of candidates including x is preferred to that set subtracting x. In addition to this, we consider three types of restrictions on candidates' preferences: low restriction, medium restriction, and high restriction. The low restriction only requires that any candidate considers herself to be elected. This is the minimal requirement to make the

[1]Barberà et al. (1991) study this model with a property known as "strategy-proofness", which requires that no voter can be better off by misrepresenting her true preference. Literature studying this model includes Shimomura (1996) and Ju (2003).

[2]Dutta et al. (2001) study *candidate stability* in the standard single-winner election model. They assume that both voters and candidates have unrestricted strict preferences over the set of candidates, and that any candidate considers herself the best in the set of candidates. They show that if the set of voters and the set of candidates are separated, then a rule satisfying *candidate stability* and respecting unanimity is only dictatorial. They also consider the case where an overlap between voters and candidates occurs.

concept of *candidate stability* meaningful. Under this restriction, a candidate can prefer another candidate to herself. This suits the situation where there are several parties, which are subgroups of the set of candidates, and a member of a party considers that the leader of the party is preferable as a winner than herself. The medium restriction requires that any candidate considers herself as the best of all candidates. Even under this restriction, a member of a party can consider that it is better that two members from the party to which she belongs are elected than if only she is elected from the party. The high restriction requires that any candidate prefers any set of candidates including herself to that excluding herself. The election of tenured faculty from members in tenure-track positions is an example that suits this requirement.

We show that under each restriction of candidates' preferences, *candidate stability* has an interesting equivalent condition. Under the low restriction, *candidate stability* is equivalent to "strong stability", which requires that the outcome when a candidate is not running equals the outcome of dropping herself. Furthermore, *candidate stability* is equivalent to the following "independence" condition. Suppose that given a set of running candidates and a preference profile, a candidate wins. Then (i) she also should win for any preference profile that is equivalent to the previous profile in the comparison between herself and the null outcome, and (ii) even if another agent drops from the candidate board, she should win.

Under the medium restriction, *candidate stability* is equivalent to "medium stability", which requires that if the outcome contains only one candidate, then the outcome when is not running is the null outcome or contains another candidate, and otherwise, it is equivalent to *strong stability.* Under the high restriction, *candidate stability* is equivalent to a weaker condition "insignificance", which requires that if a candidate is not elected, then the outcome equals that of when she does not run..

Related literature includes Berga et al. (2004, 2006) that study the same multi-winner election model. In their interpretation, voters are existing members of a society with exit options and candidates are new entrants to the society. They consider the stability of existing members while we focus on the stability of candidates. Literature studying *candidate stability* in various models includes Ehlers and Weymark (2003), Eraslan and McLennan (2004), Samejima (2005, 2007), Rodríguez-Álvarez (2006a, b) and Tanaka

(2008).

The structure of the rest of this paper is as follows. Section 2 provides the notation and basic structure of the model. Section 3 states the results. Section 4 concludes. The Appendix includes the proofs of all propositions.

2 Preliminaries

Let $C \equiv \{1, 2, \cdots, c\}$ be the set of (prospective) **candidates** with $c \geq 1$. We refer to a candidate who actually stands in the election as a **running candidate**. Let $V \equiv \{c+1, c+2, \cdots, c+v\}$ be the set of **voters** with $v \geq 1$. Throughout this paper, we assume $C \cap V = \emptyset$. For a voter or a candidate $i \in C \cup V$, let P_i denote her **preference**, which is a complete, transitive, and asymmetric binary relation over 2^C. Let D denote the domain of all preferences.

A **preference profile** is a $c+v$ tuple of preferences $P = (P_1, \cdots, P_c, P_{c+1}, \cdots, P_{c+v}) \in D^{c+v}$. For $i, j \in C \cup V$, let $(P'_i, P_{-i}) \in D^{c+v}$ denote the preference profile obtained from P by replacing P_i with P'_i, $(P''_j, P'_i, P_{\{-i,j\}}) \in D^{c+v}$ denote the profile obtained from (P'_i, P_{-i}) by replacing P_j with P''_j, and so on. Given $X \subseteq C$, let $P_i|_X$ denote the preference relation over 2^X induced by $P_i \in D$. For example, given $x, y \in C$, $P_i|_{\{x\}}$ represents the relation between $\{x\}$ and $\emptyset$ induced by P_i, and $P_i|_{\{x,y\}}$ represents the relation among $\{x, y\}$, $\{x\}$, $\{y\}$, and $\emptyset$ induced by P_i. Similarly, let $P|_X$ denote the preference profile over 2^X induced by $P \in D^{c+v}$.

A **rule** is a function $\varphi : 2^C \times D^{c+v} \to 2^C$. Following Dutta et al. (2001), a rule φ is assumed to satisfy the following three properties. First, winners should be chosen from the set of running candidates. Second, only voters' preferences matter. Third, only preferences over the running candidates matter.

Feasibility: For all $X \in 2^C$ and all $P \in D^{c+v}$, we have $\varphi(X, P) \subseteq X$.

Independence of nonvoters' preferences: For all $X \in 2^C$ and all $P, P' \in D^{c+v}$ such that for all $i \in V$, $P_i = P'_i$, we have $\varphi(X, P) = \varphi(X, P')$.

Independence of irrelevant alternatives: For all $X \in 2^C$ and all $P, P' \in D^{c+v}$ such that $P|_X = P'|_X$, we have $\varphi(X, P) = \varphi(X, P')$.

The formal definition of "candidate stability" of a rule is given as follows.

Candidate stability: For all $x \in X$, and all $P \in D^{c+v}$, we have $\varphi(C,P)\ P_x\ \varphi(C \backslash \{x\}, P)$ or $\varphi(C,P) = \varphi(C \backslash \{x\}, P)$.

We assume that all candidates' preferences satisfy the following restriction.

Separability: For all $x, y \in C$, and all $X \subseteq C \backslash \{y\}$, we have $X \cup \{y\}\ P_x\ X \iff \{y\}\ P_x\ \emptyset$.[3]

Additionally, we consider three types of restrictions on the candidates' preferences. The first only requires that any candidate considers herself as a good. The second requires that any candidate considers herself as the best of all candidates. The third requires that any candidate prefers any set of candidates including herself to that dropping herself.

Low restriction: For all $x \in C$, $\{x\}\ P_x\ \emptyset$.

Medium restriction: For all $x \in C$, $\{x\}\ P_x\ \emptyset$ and for all $y \in C \backslash \{x\}$, $\{x\}\ P_x\ \{y\}$.

High restriction: For all $x \in C$ and all $X, Y \subseteq C$ such that $x \in X$ and $x \notin Y$, $X\ P_x\ Y$.

Note that we need not any condition to voters' preferences. In the following section, we investigate how *candidate stability* works in each restriction for candidates' preferences.

3 The Results

3.1 The low restriction case

The next property states that the outcome when a candidate is not running equals the outcome of dropping herself. Proposition 1 states that in the low restriction case, *candidate stability* is equivalent to this property.

Strong stability: For all $x \in X$ and all $P \in D^{c+v}$, we have $\varphi(C \backslash \{x\}, P) = \varphi(C,P) \backslash \{x\}$.

[3] Along with separability, "additivity" is also a popular restriction to preferences. Additivity is stronger than separability and requires that preferences have additive numerical representations. See Barberà et al. (1991) and Berga et al. (2004) for the precise definition of additivity and the relationship between separability and additivity. All propositions in this paper except Proposition 3 hold for additive preferences. See Footnote 6 in the Appendix for the explanation why Proposition 3 does not hold for additive preferences.

Proposition 1. *Under the low restriction to candidates' preferences, a rule satisfies candidate stability if and only if it satisfies strong stability.*

The intuition behind the proof is as follows. Under the low restriction, there exists a candidate who prefers another candidate to win no matter whether the other candidates, including herself, are elected. There also exists a candidate who prefers another candidate not to win no matter whether the other candidates, including herself, are elected. If *strong stability* is violated, a candidate with such a preference has a chance to be better off by not running. This violates *candidate stability.*

From Proposition 1, we derive that *candidate stability* is equivalent to the following independence condition. Suppose that given a set of running candidates and a preference profile, a candidate wins. Then (i) she also should win for any preference profile that is equivalent to the previous profile in the comparison between herself and the null outcome, and (ii) moreover if another agent drops from the candidate board, she should win.

Independence: For all $x \in C$ and all $P, P' \in D^{c+v}$ such that $P|_{\{x\}} = P'|_{\{x\}}$, we have (i) $x \in \varphi(C, P) \iff x \in \varphi(C, P')$ and (ii) for all $y \in C \backslash \{x\}$, $x \in \varphi(C, P) \iff x \in \varphi(C \backslash \{y\}, P')$.

Proposition 2. *Under the low restriction to candidates' preferences, a rule satisfies candidate stability if and only if it satisfies independence.*

Note that part (i) of *independence* by itself is not a sufficient condition for *candidate stability.* An example of rules satisfying *candidate stability* under the low restriction is "voting by committees", which Barberá et al. (1991) characterize as a unique class of "strategy-proof" and "onto" rules when voters' preferences are separable. See Barberá et al. (1991) for the precise definitions of the rules and properties.[4]

3.2 The medium restriction case

Under the medium restriction, neither of Propositions 1 and 2 holds. The following property states that if the outcome contains only one candidate x,

[4] A rule of voting by committees in Barberá et al. (1991) satisfies only part (i) of *independence* as they do not care for strategic withdrawal by a candidate. To satisfy part (ii) of *independence*, we actually have to add a natural extension to voting by committees that despite the set of running candidates, the same decision should be applied to each candidate.

then the outcome when x is not running is $\emptyset$ or contains another candidate, and otherwise, it is equivalent to *strong stability*. . Proposition 3 states that in the medium restriction case, *candidate stability* is equivalent to this property.

Medium stability: For all $x \in C$ and all $P \in D^{c+v}$, (i) if $\varphi(C, P) = \{x\}$, then $\varphi(C \backslash \{x\}, P) = \emptyset$ or there exists $y \in C \backslash \{x\}$ such that $\varphi(C \backslash \{x\}, P) = \{y\}$, and (ii) otherwise, $\varphi(C \backslash \{x\}, P) = \varphi(C, P) \backslash \{x\}$.

Proposition 3. *Under the medium restriction to candidates' preferences, a rule satisfies candidate stability if and only if it satisfies medium stability.*

Under the medium restriction, if a candidate is not selected in the outcome for the original candidate board, there is a possibility that she is better off by not running. Even if she is elected, there is the possibility that she prefers another candidate not to win no matter whether other candidates, including herself, is elected and that she prefers two other candidates to be elected no matter whether other candidates, including herself, is elected. Therefore, the difference from the low restriction case occurs only when a candidate is a unique winner.

If the outcome contains at least three candidates for any preference profile when all candidates run, then *candidate stability* implies *independence* in the medium restriction case by the same proof to Proposition 2. Otherwise, there is a variety of rules satisfying *candidate stability* because of condition (i) of *medium stability*.

3.3 The high restriction case

In the high restriction case, instead of Proposition 1 or Proposition 3, the following proposition holds. The next property says that if a candidate is not elected, then the outcome equals that of when she does not run.

Insignificance: For all $x \in C$ and all $P \in D^{c+v}$, we have $x \notin \varphi(C, P) \Longrightarrow \varphi(C, P) = \varphi(C \backslash \{x\}, P)$.[5]

Proposition 4. *Under the high restriction to candidates' preferences, a rule satisfies candidate stability if and only if it satisfies insignificance.*

[5]The name of this property follows Eraslan and McLennan (2004).

The intuition behind the proof is quite simple. Under the high restriction for candidates' preferences, if a candidate is selected in the outcome, she never has an incentive not to run. Thus the only requirement for *candidate stability* is *insignificance.*

4 Concluding Remarks

In this paper, we have studied the multi-winner election model and shown the equivalent condition to *candidate stability* under each of three restrictions to candidates' preferences. As an interesting question, it remains to characterize a class of rules satisfying *candidate stability* as well as other standard properties and restrictions on the preferences of not only candidates but also voters.

Appendix

Let D_L denote the domain of all separable preferences satisfying the low restriction, D_M denote that satisfying the medium restriction, and D_H denote that satisfying the high restriction.

Proof of Proposition 1. The if part: Let φ be a rule satisfying *strong stability.* Let $x \in X$ and $P \in D_L^c \times D^v$. If $x \in \varphi(C,P)$, since x prefers herself at $P_x \in D_L$ and separability of the preference, $\varphi(C,P)\ P_x\ \varphi(C,P)\backslash\{x\}$. By *strong stability*, $\varphi(C,P)\backslash\{x\} = \varphi(C\backslash\{x\},P)$. Thus $\varphi(C,P)\ P_x\ \varphi(C\backslash\{x\},P)$. If $x \notin \varphi(C,P)$, by *strong stability*, $\varphi(C,P) = \varphi(C,P)\backslash\{x\} = \varphi(C\backslash\{x\},P)$. Hence, φ satisfies *candidate stability.*
The only if part: Let φ be a rule satisfying *candidate stability.* We show that φ satisfies *strong stability.* Note that φ is *strong stability* if and only if for all $x \in X$, and all $P \in D_L^c \times D^v$, (i) $\varphi(C,P)\backslash\{x\} \subseteq \varphi(C\backslash\{x\},P)$ and (ii) $\varphi(C\backslash\{x\},P) \subseteq \varphi(C,P)\backslash\{x\}$.

First, suppose, on the contrary, that φ does not satisfy (i). Then there exist $x \in C$, and $P \in D_L^c \times D^v$ such that $\varphi(C,P)\backslash\{x\} \nsubseteq \varphi(C\backslash\{x\},P)$. Let $y \in \varphi(C,P)\backslash(\{x\}\cup\varphi(C\backslash\{x\},P))$. Let $P'_x \in D_L$ be such that for all $Y, Z \subseteq C$ with $y \in Y$ and $y \notin Z$, $Z\ P'_x\ Y$. Then $\varphi(C\backslash\{x\},P)\ P'_x\ \varphi(C,P)$. By *independence of nonvoters' preferences*, $\varphi(C\backslash\{x\},P'_x,P_{-x})\ P'_x\ \varphi(C,P'_x,P_{-x})$. This contradicts *candidate stability.* Hence our supposition is incorrect.

Next, suppose, on the contrary, that φ does not satisfy (ii). Then there exist $x \in C$, and $P \in D_L^c \times D^v$ such that $\varphi(C \backslash \{x\}, P) \nsubseteq \varphi(C, P) \backslash \{x\}$. Let $y \in \varphi(C \backslash \{x\}, P)) \backslash \varphi(C, P)$. Let $P'_x \in D_L$ be such that for all $Y, Z \subseteq C$ with $y \in Y$ and $y \notin Z$, $Y \ P'_x \ Z$. Then $\varphi(C \backslash \{x\}, P) \ P'_x \ \varphi(C, P)$. By *independence of nonvoters' preferences*, $\varphi(C \backslash \{x\}, P'_x, P_{-x}) \ P'_x \ \varphi(C, P'_x, P_{-x})$. This contradicts *candidate stability.* Hence our supposition is incorrect.

By the above two discussions, we have that φ satisfies *strong stability.* □

Proof of Proposition 2. The if part: The part (ii) of *independence* implies *strong stability.* Thus by Proposition 1, *independence* implies *candidate stability.*

The only if part: Let φ be a rule satisfying *candidate stability.* By Proposition 1, φ satisfies *strong stability.* If $c = 1$, *independence* is trivially satisfied. Hereafter, assume $c \geq 2$. Let $x \in C$ and $P, P' \in D_L^c \times D^v$ be such that $P|_{\{x\}} = P'|_{\{x\}}$.

First, consider the case of $x \in \varphi(C, P)$. Let $y \in C \backslash \{x\}$. By *strong stability,* $\varphi(C, P) \backslash \{y\} = \varphi(C \backslash \{y\}, P)$. Thus $x \in \varphi(C \backslash \{y\}, P)$. Let $P'' \in D_L^c \times D^v$ be such that $P''|_{C \backslash \{y\}} = P|_{C \backslash \{y\}}$ and $P''|_{\{x,y\}} = P'|_{\{x,y\}}$. Then by *independence of irrelevant alternatives,* $\varphi(C \backslash \{y\}, P'') = \varphi(C \backslash \{y\}, P)$. Thus $x \in \varphi(C \backslash \{y\}, P'')$. By *strong stability,* $x \in \varphi(C, P'')$.

Let $z \in C \backslash \{x, y\}$ and $P^* \in D_L^c \times D^v$ be such that $P^*|_{C \backslash \{z\}} = P''|_{C \backslash \{z\}}$ and $P^*|_{\{x,y,z\}} = P'|_{\{x,y,z\}}$. Then, by the similar argument to the above, $x \in \varphi(C, P^*)$.

We repeat this argument for all remaining candidates $w \in C \backslash \{x, y, z\}$, until we finally obtain $x \in \varphi(C, P')$.

In the case of $x \notin \varphi(C, P)$, by the same argument to the first case, we obtain $x \notin \varphi(C, P^*)$.

Now we obtain part (i) of *independence.* By *strong stability,* part (ii) of *independence* is also obtained. □

Proof of Proposition 3. The if part. Let φ be a rule satisfying *medium stability.* Let $x \in C$ and $P \in D_M^c \times D^v$. If $\varphi(C, P) = \{x\}$, by *medium stability,* either $\varphi(C \backslash \{x\}, P) = \emptyset$ or there exists $y \in C \backslash \{x\}$ such that $\varphi(C \backslash \{x\}, P) = \{y\}$. By the medium restriction to candidates' preferences, in either case, $\varphi(C, P) \ P_x \ \varphi(C \backslash \{x\}, P)$.

If $\varphi(C, P) \neq \{x\}$, by *medium stability,* $\varphi(C \backslash \{x\}, P) = \varphi(C, P) \backslash \{x\}$. Thus by separability of the preferences, $\varphi(C, P) \ P_x \ \varphi(C \backslash \{x\}, P)$ or $\varphi(C, P) =$

$\varphi(C\backslash\{x\}, P)$.

Hence we have that φ satisfies *candidate stability.*

The only if part. Let φ be a rule satisfying *candidate stability.* Let $x \in C$ and $P \in D_M^c \times D^v$.

Case I: $x \notin \varphi(C, P)$.

Suppose, on the contrary, that $\varphi(C, P) \neq \varphi(C\backslash\{x\}, P)$. We derive a contradiction. Note that $\varphi(C, P) \neq \varphi(C\backslash\{x\}, P)$ is equivalent to $\varphi(C, P) \nsubseteq \varphi(C\backslash\{x\}, P)$ or $\varphi(C\backslash\{x\}, P) \nsubseteq \varphi(C, P)$.

If $\varphi(C, P) \nsubseteq \varphi(C\backslash\{x\}, P)$, there exists $y \in C\backslash\{x\}$ such that $y \in \varphi(C, P)\backslash\varphi(C\backslash\{x\}, P)$. Let $P'_x \in D_M$ be such that for all $X, Y \subseteq C\backslash\{x\}$ with $y \notin X$ and $y \in Y$, $X \ P'_x \ Y$. Then $\varphi(C\backslash\{x\}, P) \ P'_x \ \varphi(C, P)$. By *Independence of nonvoters' preferences,* $\varphi(C\backslash\{x\}, P'_x, P_{-x}) \ P'_x \ \varphi(C, P'_x, P_{-x})$. This contradicts *candidate stability.*

If $\varphi(C\backslash\{x\}, P) \nsubseteq \varphi(C, P)$, there exists $y \in C\backslash\{x\}$ such that $y \in \varphi(C\backslash\{x\}, P)\backslash\varphi(C, P)$. Let $P''_x \in D_M$ be such that for all $X, Y \subseteq C\backslash\{x\}$ with $y \notin X$ and $y \in Y$, $Y \ P''_x \ X$. Then $\varphi(C\backslash\{x\}, P) \ P''_x \ \varphi(C, P)$. By *Independence of nonvoters' preferences,* $\varphi(C\backslash\{x\}, P''_x, P_{-x}) \ P''_x \ \varphi(C, P''_x, P_{-x})$. This constricts *candidate stability.*

Hence, we obtain that in Case I, $\varphi(C, P) = \varphi(C\backslash\{x\}, P)$ and *medium stability* is satisfied.

Case II: $x \in \varphi(C, P)$.

First, we show that $\varphi(C, P)\backslash\{x\} \subseteq \varphi(C\backslash\{x\}, P)$. Suppose, on the contrary, that there exists $y \in C$ and $P \in D_M^c \times D^v$ such that $y \in \varphi(C, P)\backslash(\{x\} \cup \varphi(C\backslash\{x\}, P))$. We derive a contradiction. Let $P'_x \in D_M$ be such that for all $X, Y \subseteq C$ with $y \notin X$ and $y \in Y$, $X \ P'_x \ Y$. Then $\varphi(C\backslash\{x\}, P) \ P'_x \ \varphi(C, P)$. By *independence of nonvoters' preferences,* $\varphi(C\backslash\{x\}, P'_x, P_{-x}) \ P'_x \ \varphi(C, P'_x, P_{-x})$. This constricts *candidate stability.* Thus we have that φ satisfies $\varphi(C, P)\backslash\{x\} \subseteq \varphi(C\backslash\{x\}, P)$.

Then we divide Case II into two subcases.

Subcase II-i: $\{x\} \neq \varphi(C, P)$.

In this case, there exists $y \in C\backslash\{x\}$ such that $y \in \varphi(C, P)$. We show that $\varphi(C\backslash\{x\}, P) \subseteq \varphi(C, P)\backslash\{x\}$. Suppose, on the contrary, that there exists $z \in C\backslash\{x, y\}$ such that $z \in \varphi(C\backslash\{x\}, P)\backslash\varphi(C, P)$. Let $P'_x \in D_M$ be such that for all $X, Y \subseteq C$ with $y, z \in X$ and $z \notin Y$, $X \ P'_x \ Y$.[6] By *independence of*

[6]Let $x, y, z \in C$. There is x's separable preference with medium restriction P_x such that $\{y, z\} \ P_x \ \{x, y\}$. However, additive restriction excludes preferences like P_x. Thus if

nonvoters' preferences, $\varphi(C\backslash\{x\}, P'_x, P_{-x})\ P'_x\ \varphi(C, P'_x, P_{-x})$. This constricts *candidate stability*. Thus combined with $\varphi(C, P)\backslash\{x\} \subseteq \varphi(C\backslash\{x\}, P)$, we obtain $\varphi(C\backslash\{x\}, P) = \varphi(C, P)\backslash\{x\}$ and that *medium stability* is satisfied in this subcase.
Subcase II-ii: $\{x\} = \varphi(C, P)$.

We show that $\varphi(C\backslash\{x\}, P) = \emptyset$ or there exists $y \in C\backslash\{x\}$ such that $\varphi(C\backslash\{x\}, P) = \{y\}$. Since we already have that $\varphi(C, P)\backslash\{x\} \subseteq \varphi(C\backslash\{x\}, P)$, we only need to show that the number of candidates in $\varphi(C\backslash\{x\}, P)$ is less than or equal to one. Suppose, on the contrary, that there exist $y, z \in C\backslash\{x\}$ such that $y \neq z$ and $\{y, z\} \subseteq \varphi(C\backslash\{x\}, P)$. Let $P'_x \in D_M$ be such that $\{y, z\}\ P\ \{x\}$. By *independence of nonvoters' preferences*, $\varphi(C\backslash\{x\}, P'_x, P_{-x})\ P'_x\ \varphi(C, P'_x, P_{-x})$. This constricts *candidate stability*. Hence we obtain that *medium stability* is satisfied in this subcase. □

Proof of Proposition 4. the if part: Let φ be a rule satisfying *insignificance*. Let $x \in C$ and $P \in D_H^c \times D^v$. If $x \in \varphi(C, P)$, by the high restriction of the preference, $\varphi(C, P)\ P_x\ \varphi(C\backslash\{x\}, P)$. If $x \notin \varphi(C, P)$, by *insignificance*, $\varphi(C, P) = \varphi(C\backslash\{x\}, P)$. Hence, φ satisfies *candidate stability*.
the only if part: Suppose, on the contrary, that φ does not satisfy *insignificance*. Then there exist $x \in C$ and $P \in D_H^c \times D^v$ such that $x \notin \varphi(C, P)$ and $\varphi(C, P) \neq \varphi(C\backslash\{x\}, P)$. We derive a contradiction. Note that $\varphi(C, P) \neq \varphi(C\backslash\{x\}, P)$ is equivalent to $\varphi(C, P) \nsubseteq \varphi(C\backslash\{x\}, P)$ or $\varphi(C\backslash\{x\}, P) \nsubseteq \varphi(C, P)$.

If $\varphi(C, P) \nsubseteq \varphi(C\backslash\{x\}, P)$, there exists $y \in C\backslash\{x\}$ such that $y \in \varphi(C, P)\backslash\varphi(C\backslash\{x\}, P)$. Let $P'_x \in D_H$ be such that for all $X, Y \subseteq C\backslash\{x\}$ with $y \notin X$ and $y \in Y$, $X\ P'_x\ Y$. Then $\varphi(C\backslash\{x\}, P)\ P'_x\ \varphi(C, P)$. By *Independence of nonvoters' preferences*, $\varphi(C\backslash\{x\}, P'_x, P_{-x})\ P'_x\ \varphi(C, P'_x, P_{-x})$. This contradicts *candidate stability*.

If $\varphi(C\backslash\{x\}, P) \nsubseteq \varphi(C, P)$, there exists $y \in C\backslash\{x\}$ such that $y \in \varphi(C\backslash\{x\}, P)\backslash\varphi(C, P)$. Let $P''_x \in D_H$ be such that for all $X, Y \subseteq C\backslash\{x\}$ with $y \notin X$ and $y \in Y$, $Y\ P''_x\ X$. Then $\varphi(C\backslash\{x\}, P)\ P''_x\ \varphi(C, P)$. By *Independence of nonvoters' preferences*, $\varphi(C\backslash\{x\}, P''_x, P_{-x})\ P''_x\ \varphi(C, P''_x, P_{-x})$. This constricts *candidate stability*.

Hence, we have that φ satisfies *insignificance*. □

---

we impose additivity, the equivalent condition to *candidate stability* under the medium restriction is slightly weaker than *medium stability*.

References

Barberà, S., H. Sonnenschein, L. Zhou, 1991. Voting by committees. Econometrica 59 (3), 595-609.

Berga, D., G. Bergantiños, J. Massó, A. Neme, 2004. Stability and voting by committees with exit. Social Choice and Welfare 23 (2), 229-247.

Berga, D., G. Bergantiños, J. Massó, A. Neme, 2006. On exiting after voting. International Journal of Game Theory 34 (1), 33-54.

Dutta, B., M. O. Jackson, M. Le Breton, 2001. Strategic candidacy and voting procedures. Econometrica 69 (4), 1013-1037.

Ehlers, L., J. A. Weymark, 2003. Candidate stability and nonbinary social choice. Economic Theory 22 (2), 233-243.

Eraslan, H., A. McLennan, 2004. Strategic candidacy for multivalued voting procedures. Journal of Economic Theory 117 (1), 29-54.

Ju, B.-G., 2003. A characterization of strategy-proof voting rules for separable weak orderings. Social Choice and Welfare 21 (3), 469-499.

Rodríguez-Álvarez, C, 2006a. Candidate stability and probabilistic voting procedures. Economic Theory 27 (3), 657-677.

Rodríguez-Álvarez, C, 2006b. Candidate stability and voting correspondences. Social Choice and Welfare 27 (3), 545-570.

Samejima, Y, 2005. Strategic candidacy, monotonicity, and strategy-proofness. Economics Letters 88 (2), 190-195.

Samejima, Y, 2007. Strategic candidacy and single-peakedness. Japanese Economic Review 58 (4), 423-442.

Shimomura, K.-I., 1996. Partially efficient voting by committees. Social Choice and Welfare 13 (3), 327-342.

Tanaka, Y, 2008. Undecidability of the existence of dictator for strongly candidate stable voting procedures in an infinite society and Cantor's diagonal argument. Computational and Applied Mathematics 27 (3), 251-268.