Distributed Route Selection under Consideration of Time Dependency among Agents' Road Usage for Vehicular Networks

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SUMMARY Traffic congestion in road networks has been studied as the congestion game in game theory. In the existing work, the road usage by each agent was assumed to be static during the whole time horizon of the agent's travel, as in the classical congestion game. This assumption, however, should be reconsidered because each agent sequentially uses roads composing the route. In this paper, we propose a multi-commodity distributed route selection scheme based on a gradient descent method considering the time-dependency among agents' road usage for vehicular networks. In the proposed scheme, each agent has multiple route candidates and iteratively calculates the optimal route choice probability of each route candidate using the gradient descent method and time-dependent flow, which is estimated based on the other agents' route choice probabilities shared through communication networks. Each agent finally selects one route according to the optimal route choice probabilities. Through the numerical results and the simulation, we show that the proposed scheme can improve the actual travel time by 5.1% compared with the static-flow based approach.

key words: *Multi-commodity distributed route selection; time-dependent flow; gradient descent method; road network ; vehicular network*

1. Introduction

It has been well-known that the traffic congestion problem can be modeled as a congestion game in game theory by regarding roads as resources. Since the route selection of one agent results in the use of roads composing the corresponding route, the route selection of all agents determines the assignment of agents to roads, which will finally determine the travel time of each agent. It is rational for each agent to select a route that seems to have the minimum travel time from its route candidates. Such route selection is called selfish routing and results in a Wardrop equilibrium where each agent cannot reduce its travel time by changing the route [1, 2].

Since the classical congestion game assumes that each player can select one or more resources at the same time, its straightforward extension to the traffic congestion problems also assumes the static flow where each agent simultaneously uses the roads composing of its route during the whole time horizon of its travel [2, 3]. However, in case of the road networks, each agent moves along with the route, which indicates that the agent will sequentially use the roads in the route.

In this paper, we propose a multi-commodity distributed route selection scheme considering the time dependency among agents' road usage for vehicular networks. The proposed scheme comprises of the following two procedures. First one is the estimation of the time-dependent competitive relationship among agents considering a time dependency among agents' road usage. Second one is the distributed route selection based on a gradient descent method with the time-dependent flow information, which is an expanded version of the existing scheme in [4]. Through the numerical and simulation results, we show that the proposed scheme can improve the actual travel time by 5.1% compared with the existing scheme [4], with the help of the accurate estimation of the congestion levels.

The rest of the manuscript is organaized as follows. Section 2 gives related work. We introduce the distributed route selection scheme considering the time dependency among agents' road usage in Section 3. Section 4 demonstrates the performance of the proposed scheme through the numerical and simulation results. Section 5 gives the conclusion and the future work.

2. Related Work

With the help of the classical congestion game, Lim and Rus proposed the multi-commodity distributed route selection scheme under the assumption of the static flow [4]. This scheme interprets the route choice probability of each agent as the fractional flow under the assumption that the number of agents is sufficiently large. As a result, the flow on a road can be expressed by the agents' probabilistic occupation [5–7]. Each agent autonomously calculates the optimal route choice probabilities for its route candidates such that its travel time will be minimized.

Ford and Fulkerson first introduced the concept of the flow over time and the time-expanded network [8, 9]. In contrast to the static flow, the flow over time assumes that the flow on a road dynamically changes. The time-expanded network contains one copy of the static network for each discrete time step. The flow over time in the static network can be interpreted as the static flow in the time-expanded network. Such time-expanded network uses the fixed travel time of the road under the assumption that the capacity of the edge limits the flow into the edge at each time step [10]. Köhler et al. proposed a time-expanded graph for the flow-dependent transit time [11].

Some existing studies proved the existence of Nash equilibria for flow over time [12, 13]. Anshelevich and Ukkusuri showed the existence of Nash equilibria in single-source

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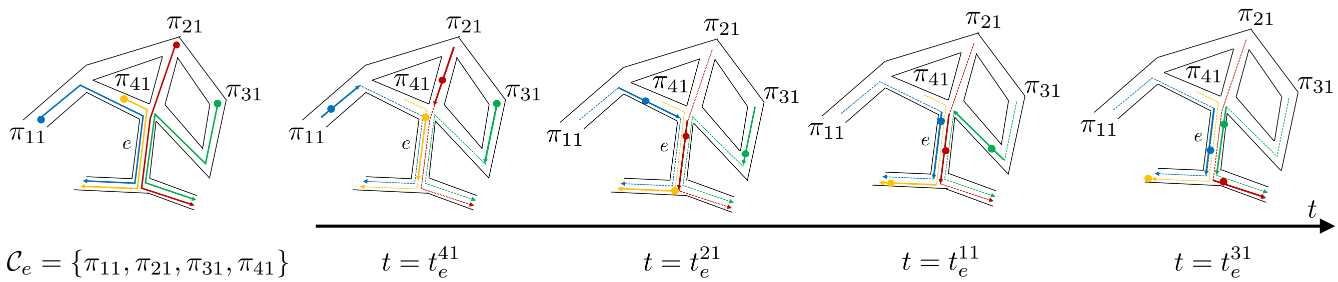

(a) Static flow case

(b) Time-dependent flow case

Fig. 1 Probabilistic occupation of road e by corresponding agent' routes.

single-sink network where the traffic obeys the first-in-first-out (FIFO) policy [12]. Koch and Skutella introduced the congestion game with flow over time by using a deterministic queuing model [13]. In the deterministic queuing model, the traffic is regarded as the flow particles (infinitesimal flow units). The travel time of each road consists of the fixed transit time plus the waiting time. The fixed transit time means the time that a flow particle needs to travel from the tail to the head of the road. If the traffic demand exceeds the road capacity, flow particles queue up at the end of the road in the FIFO manner. In this paper, we consider the time-dependent competitive relationship among agents in a different way compared with the time-expanded network and the deterministic queuing model.

3. Distributed Route Selection under Consideration of Time Dependency among Agents' Road Usage

3.1 Preliminaries

$G = (\mathcal{V}, \mathcal{E})$ denotes a graph representing the internal structure of the road network, where $\mathcal{V}$ denotes a set of vertices (i.e., intersections) and $\mathcal{E}$ denotes a set of edges (i.e., roads). There are N ($N > 0$) agents (e.g., vehicles) in the road network and $\mathcal{N} = \{1, \ldots, N\}$ denotes a set of agents. Each agent $i \in \mathcal{N}$ first calculates K_i ($K_i > 0$) candidate routes $\boldsymbol{\pi}_i = (\pi_{i1}, \ldots, \pi_{iK_i})$ where route π_{ik} is the agent i's kth route candidate, which is a vector of edges composing the corresponding route. Let $\mathcal{K}_i = \{1, \ldots, K_i\}$ be a set of the corresponding route indices.

Next, each agent $i \in \mathcal{N}$ calculates route choice probabilities $\boldsymbol{p}_i = (p_{i1}, \ldots, p_{iK_i})^\top$ where p_{ik} ($0 \leq p_{ik} \leq 1$) denotes the probability that agent i selects the kth route. Note that $\sum_{k \in \mathcal{K}_i} p_{ik} = 1$ and $\boldsymbol{p}_i$ can also be regarded as the mixed strategy in game theory [14]. We assume that each agent $i \in \mathcal{N}$ collects the route choice probabilities $\boldsymbol{p}_j$ of competing agents $j \in \mathcal{N}_i$ through communication networks (e.g., cellular networks and vehicular networks). (The definition of $\mathcal{N}_i$ will be given in Section 3.2.) Then, each agent i calculate $\boldsymbol{p}_i$ using a gradient descent method and $\boldsymbol{p}_j$ ($j \in \mathcal{N}_i$). Finally, each agent i selects a certain route, π_{ik^*}, according to the route choice probabilities $\boldsymbol{p}_i$. We assume that the route calculation is periodically conducted at a certain interval I_{M} ($I_{\mathrm{M}} > 0$) to suppress the estimation error and adapt to environmental changes, e.g., new agent arrivals.

If the number of agents is large, we can regard the route choice probability p_{ik} ($i \in \mathcal{N}, k \in \mathcal{K}_i$) as the fractional flow as in [5, 15, 16]. Therefore, the flow on a road can also be interpreted as the probabilistic occupation by the corresponding agent's route. In the classical congestion game, the probabilistic occupation of a road is assumed to be static during the whole time horizon of the agent's travel [5–7]. Fig 1 illustrates an example of the probabilistic occupation of road e by four agents' routes (i.e., $\{\pi_{11}, \pi_{21}, \pi_{31}, \pi_{41}\}$). In case of the static flow assumption (Fig. 1a), the flow on road e, f_e, is defined as the sum of the corresponding route choice probabilities:

$$f_e = \sum_{\pi_{jl} \in C_e} p_{jl}, \tag{1}$$

where C_e denotes a set of each agent's route that includes road e.

3.2 Time Dependency among Agents' Road Usage

Since each agent travels along a route, its probabilistic occupation of each road in the route will sequentially happen as shown in Fig. 1b. In this case, the static flow assumption, where all the roads composing the route are simultaneously and continuously used, should be reconsidered. When the agent 1 following the route π_{11} just enters the road e at the time t_e^{11}, the agent 2 following the route π_{21} is only the leading competitor on the road e. Then, the agent 1 will have the agent 3 as its follower on the road e at its arrival on the road e at the time t_e^{31}. Note that the agent 1's movement on the road e will be affected only by its leading agent, i.e., agent 2.

As a result, we can define a set of time-dependent competitive routes of the road e for the agent i following the route π_{ik}:

$$C_e^{ik} = \{\pi_{jl} \in C_e \mid t_e^{jl} \leq t_e^{ik} \leq t_e^{jl} + \widetilde{t}_e\}. \tag{2}$$

t_e^{ik} denotes the inflow time when the agent i following π_{ik} enters road e. $\widetilde{t}_e$ denotes the estimated travel time on the road e. (The estimation method will be discussed in the next section.) $t_e^{jl} + \widetilde{t}_e$ denotes the outflow time when the agent j following π_{jl} exits the road e. The condition $t_e^{jl} \leq t_e^{ik} \leq$

$t_e^{jl} + \widetilde{t}_e$ guarantees that the agent j following π_{jl} leads the agent i following π_{ik} on the road e, and thus becomes the competitor. Note that this model assumes that all agents obey the FIFO policy and C_e^{ik} does not change until the agent i following π_{ik} exits the road e. Using C_e^{ik}, we can express the time-dependent flow on the road e for the agent i's route π_{ik}.

$$f_e^{ik} = \sum_{\pi_{jl} \in C_e^{ik}} p_{jl}. \tag{3}$$

Furthermore, each agent $i \in \mathcal{N}$ collects the route choice probabilities $\boldsymbol{p}_j$ of competitors $j \in \mathcal{N}_i$ through communication networks. Note that $\mathcal{N}_i = \{j \in \mathcal{N} \setminus \{i\} \mid \exists l \in \mathcal{K}_j,\ \pi_{jl} \in \cup_{k \in \mathcal{K}_i} C_e^{ik}\}$.

3.3 Distributed Route Selection under Consideration of Time-Dependent Road Usage

The cost of route π_{ik} for agent i can be expressed by the sum of cost of each road along route π_{ik}.

$$c_{ik} = \sum_{\forall e \in \pi_{ik}} c_e(f_e^{ik}),$$

where $c_e(\cdot)$ denotes the cost of the road e under the flow f_e^{ik}, which is a non-decreasing function. From Eqs. (2) and (3), we should note here that $c_e(\cdot)$ depends on $\widetilde{t}_e$. This is a kind of the chicken or egg situations, and thus it is hard to obtain accurate $\widetilde{t}_e$ for each road $e \in \mathcal{E}$ before calculating the path cost c_{ik} $(i \in \mathcal{N}, k \in \mathcal{K}_i)$. In this paper, we simply regard $\widetilde{t}_e$ as the lower bound of travel time on the road e, $\underline{t}_e$, which is the travel time without any congestion on the road e. In Section 4.2, we will show this simple assumption contributes to the congestion alleviation but we also plan to apply more sophisticated estimation methods [17, 18].

We assume that each agent $i \in \mathcal{N}$ measures the goodness of the current route choice probabilities $\boldsymbol{p}_i$ based on the local cost V_i [4]:

$$V_i = \sum_{k \in \mathcal{K}_i} p_{ik} c_{ik} - c_{id_i} = \sum_{k \in \mathcal{K}_i} p_{ik}(c_{ik} - c_{id_i}), \tag{4}$$

where $d_i = \arg\min_{k \in \mathcal{K}_i} c_{ik}$. Eq. (4) means the difference between the expected path cost under $\boldsymbol{p}_i$ and the minimum path cost. It is rational for each agent i to aim at adjusting the route choice probabilities $\boldsymbol{p}_i$ such that V_i approaches to 0. $V_i = 0$ leads to the two conditions of Wardrop equilibrium [4]. The first condition means that each agent $i \in \mathcal{N}$ selects the minimum-cost path while the second one indicates that unselected paths have equal or larger cost than the minimum cost. The global cost V is defined as the sum of the local cost V_i of all agents $i \in \mathcal{N}$ [4]:

$$V = \sum_{i \in \mathcal{N}} V_i.$$

When each agent i aims to adjust $\boldsymbol{p}_i$ such that V_i approaches to 0, V also converges to 0. As a result, Wardrop equilibrium is achieved among all agents.

4. Simulation Results

4.1 Evaluation Model

To evaluate the fundamental characteristic of the proposed scheme, we use a grid road network consisting of 50×50 nodes (intersections). There are fifty agents ($N = 50$) and each agent $i \in \mathcal{N}$ travels from node $(i, 1)$ to node $(i, 50)$. Note that $(1, 1)$ (resp. $(50, 50)$) is the left-top (resp. right-bottom) node of the grid network. We assume that travel time of each road $e \in \mathcal{E}$ follows the BPR function [19], i.e., $t_e(f_e) = \underline{t}_e(1 + \alpha(f_e/c_e)^\beta)$ where $\underline{t}_e$ denotes the travel time without any congestion on the road e, and c_e denotes the capacity of the road e. α and β represent the degree of the traffic congestion. For each road e, we randomly set $\underline{t}_e$ (resp. c_e) in the range of $[0, 1]$ (resp. $[2, 4]$). We also use $\alpha = 0.15$ and $\beta = 4$.

Each agent $i \in \mathcal{N}$ obtains $K_i = 5$ route candidates $\boldsymbol{\pi}_i$ according to the following procedure. Each agent $i \in \mathcal{N}$ first calculates the shortest route from the origin, i.e., node $(i, 1)$, to the destination, i.e., node $(i, 50)$, when the flow of the agent i only exists, i.e., $t_e(1) = \underline{t}_e(1 + \alpha(1/c_e)^\beta)$. Next, it obtains the second route candidate by calculating the shortest route from the origin to the destination under the assumption that the predefined number of road segments, i.e., 30, in the shortest route are unavailable. By repeating this procedure, each agent $i \in \mathcal{N}$ obtains K_i route candidates $\boldsymbol{\pi}_i$, which are exclusive to each other as much as possible.

As for evaluation criteria, we use the estimated travel time and the actual one. The estimated travel time for agent i, $\widetilde{T}_i$, is defined as the weighted sum of the travel time corresponding each route π_{ik}.

$$\widetilde{T}_i = \sum_{k \in \mathcal{K}_i} p_{ik}^* \sum_{e \in \pi_{ik}} t_e(f_e^{ik}),$$

where the weights are given by $\boldsymbol{p}_i^*$, which is derived by the gradient descent method.

To obtain the actual travel time T_i, i.e., the elapsed time between the departure from the origin and the arrival at the destination, for each agent $i \in \mathcal{N}$, we implemented a Java simulator. In what follows, we assume that the control interval I_{M} is sufficiently large and the route calculation is conducted once at the beginning of the simulation. Each agent $i \in \mathcal{N}$ first calculates the route choice probabilities $\boldsymbol{p}_i^*$, and then selects one of the candidates, k_i^*, according to $\boldsymbol{p}_i^*$. We define the deterministic version of $\boldsymbol{p}_i^*$ as $\widehat{\boldsymbol{p}}_i = (\widehat{p}_{i1}, \ldots, \widehat{p}_{iK_i})^\top$. $\widehat{\boldsymbol{p}}_i$ is a vector of size K_i where k_i^*th element is set to be 1 and the remaining elements are set to be 0.

We use the following congestion model in the simulator. Suppose that three agents i $(i = 1, 2, 3)$ selecting the route π_{i1} travel a road e in the same direction and in this order. We obtain $f_e^{11} = \widehat{p}_{11} = 1$, $f_e^{21} = \widehat{p}_{11} + \widehat{p}_{21} = 2$, and $f_e^{31} = \widehat{p}_{11} + \widehat{p}_{21} + \widehat{p}_{31} = 3$ from Eq. (3). As a result, the agent

Table 1 Estimated travel time and actual travel time

Scheme	$\widetilde{T}_i$ [min]			T_i [min]		
	avg.	max.	std.	avg.	max.	std.
Proposed scheme	23.1	25.1	1.32	22.3	24.2	1.05
Conventional scheme [4]	28.0	29.8	0.87	23.5	25.9	1.17

i moves as the speed of $v_i = d_e/t_e(f_e^{i1})$ where d_e is the length of the road e. When the agent 1 exits the road e, the time-dependent flow for the agents 2 (resp. 3) is updated to $f_e^{21} = \widehat{p}_{21} = 1$ (resp. $f_e^{31} = \widehat{p}_{21} + \widehat{p}_{31} = 2$), and thus the corresponding speed also changes.

For comparison purpose, we also use the conventional scheme, which is the distributed route selection scheme based on the classical congestion game [4], i.e., $C_e^{ik} = C_e$. In what follows, we show the average of 100 independent experiments.

4.2 Fundamental Results

Table 1 shows the estimated travel time $\widetilde{T}_i$ and the actual one T_i for both schemes in terms of the average and maximum values. We first focus on the difference between the estimated travel time and actual one for each scheme. We observe that the proposed scheme can more accurately estimate the travel time than the conventional scheme. In particular, the relative estimation error, i.e., $(\widetilde{T}_i - T_i)/T_i$, of the proposed scheme is 0.036 (resp. 0.037) in case of the average (resp. maximum) travel time, which is much smaller than that of the conventional scheme (i.e., 0.19 (resp. 0.15) in case of the average (resp. maximum) travel time).

Next, we focus on the performance difference between the proposed scheme and the conventional scheme. We observe that the proposed scheme can improve the average (resp. maximum) actual travel time by 5.1% (resp. 6.6%) compared with the conventional scheme. The static flow assumption used in the conventional scheme considers the worst congestion case while the time-dependent flow assumption in the proposed scheme seems to succeed in estimating more possible congestion level of each road.

5. Conclusion

In the classical routing game, all the roads composing the route are assumed to be used simultaneously and continuously. However, this assumption should be reconsidered since the congestion level would change over time. In this paper, we have proposed a multi-commodity distributed route selection scheme based on a gradient descent method considering time dependency among agents' road usage. In the proposed scheme, each agents calculates the route choice probabilities by using the estimated time-dependent flow on each road in the distributed manner. Through the numerical results and the simulation results, we have shown that the proposed scheme can improve the actual travel time by 5.1%.

In future work, we plan to investigate how the proposed scheme can improve the actual travel time by controlling the interval I_{M}, which will contribute to increase the estimation accuracy at the expense of the computational complexity. In addition, we also consider to apply the existing predictive traffic congestion models to estimate the time-dependent flow more accurately. Finally, we will conduct the mathematical analysis of the convergence property.

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