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Multi-Agent Distributed Route Selection under Consideration of Time Dependency among Agents' Road Usage for Vehicular Networks*

Takanori HARA[†a)], **Masahiro SASABE**[†b)], ***Members**, and* **Shoji KASAHARA**[†c)], ***Fellow***

SUMMARY Traffic congestion in road networks has been studied as the congestion game in game theory. In the existing work, the road usage by each agent was assumed to be static during the whole time horizon of the agent's travel, as in the classical congestion game. This assumption, however, should be reconsidered because each agent sequentially uses roads composing the route. In this paper, we propose a multi-agent distributed route selection scheme based on a gradient descent method considering the time-dependency among agents' road usage for vehicular networks. The proposed scheme first estimates the time-dependent flow on each road by considering the agents' probabilistic occupation under the first-in-first-out (FIFO) policy. Then, it calculates the optimal route choice probability of each route candidate using the gradient descent method and the estimated time-dependent flow. Each agent finally selects one route according to the optimal route choice probabilities. We first prove that the proposed scheme can exponentially converge to the steady-state at the convergence rate inversely proportional to the product of the number of agents and that of individual route candidates. Through simulations under a grid-like network and a real road network, we show that the proposed scheme can improve the actual travel time by 5.1% and 2.5% compared with the conventional static-flow based approach, respectively.

key words: *Multi-agent distributed route selection; time-dependent flow; gradient descent method; road network; vehicular networks*

1. Introduction

It has been well-known that the traffic congestion problem can be modeled as a congestion game in game theory by regarding roads as resources. Since the route selection of one agent results in the use of roads composing the corresponding route, the route selection of all agents determines the assignment of agents to roads, which will finally determine the travel time of each agent. It is rational for each agent to select a route that seems to have the minimum travel time from its route candidates. Such route selection is called selfish routing and results in a Wardrop equilibrium where each agent cannot reduce its travel time by changing the route [2], [3].

Since the classical congestion game assumes that each player can select one or more resources at the same time, its straightforward extension to the traffic congestion problems also assumes the static flow where each agent simultaneously uses the roads composing of its route during the whole time horizon of its travel [3], [4]. However, in case of the road networks, each agent moves along with the route, which indicates that the agent will sequentially use the roads in the route.

Ford and Fulkerson introduced the concept of flow over time to deal with the congestion over time [5], [6]. They also proposed a time-expanded network that contains one copy of the static network for each discrete time step. The time-expanded network enables us to use the algorithms designed for the static flow assumption at the expense of the enormous network size. Koch and Skutella studied the characteristics of Nash equilibria and the price of anarchy for the flow over time by using the deterministic queuing model [7]. The deterministic queuing model is also used in other studies to investigate the impact of the competition on the network efficiency [8]–[11].

Lim and Rus proposed a distributed route selection scheme for each agent under the assumption of the classical congestion game [12]. In [12], each agent in the road network autonomously calculates the route choice probabilities for its route candidates by using a gradient descent method such that its expected travel time is minimized. This scheme regards the route choice probability of each agent as the fractional flow under the assumption that the number of agents on the road network is sufficiently large. As a result, the flow on a road can be expressed by the probabilistic occupation for agents [13]–[15].

In this paper, we propose a multi-agent distributed route selection scheme considering the time dependency among agents' road usage. The proposed scheme comprises of the following two procedures. First one is the estimation of the time-dependent competitive relationship among agents considering a time dependency among agents' road usage. Second one is the distributed route selection based on a gradient descent method with the time-dependent flow information, which is an expanded version of the existing scheme in [12].

The main contributions of the paper are as follows:

1. We propose a multi-agent route selection considering the time dependency among agents' road usage in a distributed manner.

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†The authors are with the Division of Information Science, Nara Institute of Science and Technology, 8916-5 Takayama-cho, Ikoma, Nara, 630-0192, Japan.

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a) E-mail: hara@ieee.org
b) E-mail: m-sasabe@ieee.org
c) E-mail: kasahara@ieee.org

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2. Through simulations under a grid-like road network and a real one, we show that the proposed scheme can improve the actual travel time by 5.1% and 2.5% compared with the existing scheme [12], with the help of the accurate estimation of the congestion levels.
3. We prove that the proposed scheme can exponentially converge to the steady-state at the convergence rate inversely proportional to the product of the number of agents and that of individual route candidates. The convergence property is also confirmed through numerical results.

The rest of the paper is organized as follows. Section 2 gives related work. We introduce the multi-agent distributed route selection scheme considering the time dependency among agents' road usage in Section 3. Section 4 demonstrates the performance of the proposed scheme through the numerical and simulation results. Section 5 gives the conclusion and the future work.

2. Related Work

With the help of the classical congestion game, Lim and Rus proposed the multi-agent distributed route selection scheme under the assumption of the static flow [12]. This scheme interprets the route choice probability of each agent as the fractional flow under the assumption that the number of agents is sufficiently large. As a result, the flow on a road can be expressed by the agents' probabilistic occupation [13]–[15]. Each agent autonomously calculates the optimal route choice probabilities for its route candidates such that its travel time will be minimized. Note that the authors proved that each agent's rational route selection results in the Wardrop equilibrium.

Ford and Fulkerson first introduced the concept of the flow over time and the time-expanded network [5], [6]. In contrast to the static flow, the flow over time assumes that the flow on a road dynamically changes. The time-expanded network contains one copy of the static network for each discrete time step. The flow over time in the static network can be interpreted as the static flow in the time-expanded network. Such time-expanded network uses the fixed travel time of the road under the assumption that the capacity of the edge limits the flow into the edge at each time step [16]. Köhler et al. proposed a time-expanded graph for the flow-dependent transit time [17].

Some existing studies proved the existence of Nash equilibria for flow over time [7], [8]. Anshelevich and Ukkusuri showed the existence of Nash equilibria in single-source single-sink network where the traffic obeys the first-in-first-out (FIFO) policy [8]. Koch and Skutella introduced the congestion game with flow over time by using a deterministic queuing model [7]. In the deterministic queuing model, the traffic is regarded as the flow particles (infinitesimal flow units). The travel time of each road consists of the fixed transit time plus the waiting time. The fixed transit time means the time that a flow particle needs to travel from the tail to the head of the road. If the traffic demand exceeds the road capacity, flow particles queue up at the end of the road in the FIFO manner. In [9]–[11], the authors studied the complexity properties under the competitive routing over time, Braess's paradox over time, and Stackelberg strategy over time, respectively. In this paper, we consider the time-dependent competitive relationship among agents in a different way compared with the time-expanded network and the deterministic queuing model.

There were several studies on predictive traffic congestion model [18], [19]. Kong et al. proposed an approach for urban traffic congestion prediction and estimation by using the floating car trajectory data [18]. Fouladgar et al. proposed an urban traffic congestion prediction scheme using a deep neural network for modeling traffic flow [19]. Such predictive traffic congestion models would help the proposed scheme to estimate the time-dependent flow more accurately.

The congestion-aware routing using traffic data was also studied [20], [21]. Afshar-Nadjafi and Afshar-Nadjafi formulated a mixed integer problem for time-dependent vehicle routing to minimize the travel cost and proposed the heuristic algorithm [20]. Rossi et al. addressed the congestion-aware routing for autonomous vehicles and proposed an optimization method to minimize the congestion by allocating empty vehicles to non-crowded routes in a capacitated road network [21].

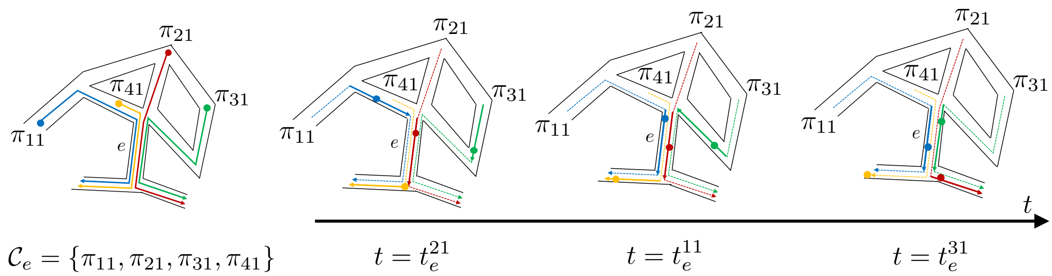
3. Distributed Route Selection under Consideration of Time Dependency among Agents' Road Usage

In this section, we propose a multi-agent distributed route selection scheme considering the time dependency of the agents' road usage, which is an extended version of the existing scheme [12].

3.1 Preliminaries

$G = (\mathcal{V}, \mathcal{E})$ denotes a graph representing the internal structure of the road network, where $\mathcal{V}$ denotes a set of vertices (i.e., intersections) and $\mathcal{E}$ denotes a set of edges (i.e., roads). There are N ($N > 0$) agents (e.g., vehicles) in the road network and $\mathcal{N} = \{1, \ldots, N\}$ denotes a set of agents. Each agent $i \in \mathcal{N}$ first calculates K_i ($K_i > 0$) candidate routes $\boldsymbol{\pi}_i = (\pi_{i1}, \ldots, \pi_{iK_i})$ where route π_{ik} is the agent i's kth route candidate, which is a vector of edges consisting of the corresponding route. Let $\mathcal{K}_i = \{1, \ldots, K_i\}$ be a set of the corresponding route indices.

Next, each agent $i \in \mathcal{N}$ calculates route choice probabilities $\boldsymbol{p}_i = (p_{i1}, \ldots, p_{iK_i})^\top$ where p_{ik} ($0 \le p_{ik} \le 1$) denotes the probability that agent i selects the kth route. Note that $\sum_{k \in \mathcal{K}_i} p_{ik} = 1$ and $\boldsymbol{p}_i$ can also be regarded as the mixed strategy in game theory [22]. We assume that each agent $i \in \mathcal{N}$ collects the route choice probabilities $\boldsymbol{p}_j$ of competing agents $j \in \mathcal{N}_i$ through communication networks e.g., cellular networks and vehicular networks. (The definition of $\mathcal{N}_i$ will be given in Section 3.2.) Then, each agent i calculate $\boldsymbol{p}_i$ using a gradient descent method and $\boldsymbol{p}_j$



(a) Static flow case. (b) Time-dependent flow case.

Fig. 1: Probabilistic occupation of road e by corresponding agent' routes.

Table 1: Notations.

Symbol	Description
G	A directed graph representing road network $G = (\mathcal{V}, \mathcal{E})$
$\mathcal{V}$	A set of vertices
$\mathcal{E}$	A set of edges
$\mathcal{N}$	A set of agents, $N = \|\mathcal{N}\|$
$\boldsymbol{\pi}_i$	A vector of route candidates for agent i, $(\pi_{i1}, \ldots, \pi_{iK_i})$
$\boldsymbol{p}_i$	A vector of route choice probabilities for agent i, $(p_{i1}, \ldots, p_{iK_i})$
$\mathcal{K}_i$	A set of indices of routes for agent i, $\{1, \ldots, K_i\}$
C_e	A set of routes that include road e
C_e^{ik}	A set of routes that include road e where the probabilistic occupation by agents following that route
f_e	Flow of road e
$c_e(\cdot)$	Cost of road e
$c_i k$	Cost of route π_{ik}
$\tilde{t}_e$	Estimated travel time of road e
$\underline{t}_e$	Travel time without any congestion on road e
t_e^{ik}	Inflow time when the agent i following π_{ik} enters road e
f_e^{ik}	The time-dependent flow on the road e for the agent i's route π_{ik}
k_i^*	Index of the minimum cost route for agent i
$\mathcal{N}_i$	Competitors of agent i
I_{M}	Calculation interval
V_i	Local cost
V	Global cost
w_{ik}	Local cost increase of the route π_{ik}
y_{ik}	Processed version of w_{ik}
$\mathbf{1}_{K_i}^{\mathrm{T}}$	Column vector of size K_i with all elements set to be 1
$\boldsymbol{e}_{k_i^*}$	Column vector of size K_i with k_i^*th element set to be 1
γ	Learning rate
τ	Iteration time of algorithm execution
v_i	Moving speed for the agent i
d_e	Length of road e
$t_e(\cdot)$	Travel time of road e
c_e	Capacity of road e
α, β	Parameter representing the degree of the traffic congestion
$\hat{\boldsymbol{p}}_i^*$	Deterministic version of $\boldsymbol{p}_i^*$ where k_i^*th element is set to be 1 and the remaining elements are set to be 0
T_i	Actual travel time of the agent i
$\tilde{T}_i$	Estimated travel time of the agent i

$(j \in \mathcal{N}_i)$. Finally, each agent i selects a certain route, π_{ik^*}, according to the route choice probabilities $\boldsymbol{p}_i$. We assume that the route calculation is periodically conducted at a certain interval I_{M} $(I_{\mathrm{M}} > 0)$ to suppress the estimation error and adapt to environmental changes, e.g., new agent arrivals.

If the number of agents is large, we can regard the route choice probability p_{ik} $(i \in \mathcal{N}, k \in \mathcal{K}_i)$ as the fractional flow as in [13], [23], [24]. Therefore, the flow on a road can also be interpreted as the probabilistic occupation by the corresponding agent' route. In the classical congestion game, the probabilistic occupation of a road is assumed to be static during the whole time horizon of the agent's travel [13]–[15]. Fig 1 illustrates an example of the probabilistic occupation of road e by four agents' routes (i.e., $\{\pi_{11}, \pi_{21}, \pi_{31}, \pi_{41}\}$). In case of the static flow assumption (Fig. 1a), the flow on road e, f_e, is defined as the sum of the corresponding route choice probabilities:

$$f_e = \sum_{\pi_{jl} \in C_e} p_{jl},$$

where C_e denotes a set of each agent's route that includes road e. Table 1 summarizes the notations used in this paper.

3.2 Time Dependency among Agents' Road Usage

Since each agent travels along a route, its probabilistic occupation of each road composing that route will sequentially happen as shown in Fig. 1b. In this case, the static flow assumption, where all the roads composing the route are simultaneously and continuously used, should be reconsidered. When the agent 1 following the route π_{11} just enters the road e at the time t_e^{11}, the agent 2 following the route π_{21} is only the leading competitor on the road e. Then, the agent 1 will have the agent 3 as its follower on the road e at its arrival on the road e at the time t_e^{31}. Note that the agent 1's movement on the road e will be affected only by its leading agent, i.e., agent 2.

As a result, we can define a set of time-dependent competitive routes of the road e for the agent i following the route π_{ik}:

$$C_e^{ik} = \{\pi_{jl} \in C_e \mid t_e^{jl} \le t_e^{ik} \le t_e^{jl} + \widetilde{t}_e\}. \quad (1)$$

t_e^{ik} denotes the inflow time when the agent i following π_{ik} enters road e. $\widetilde{t}_e$ denotes the estimated travel time on the road e. (The estimation method will be discussed in the next section.) $t_e^{jl} + \widetilde{t}_e$ denotes the outflow time when the agent j following π_{jl} exits the road e. The condition $t_e^{jl} \le t_e^{ik} \le t_e^{jl} + \widetilde{t}_e$ guarantees that the agent j following π_{jl} leads the agent i following π_{ik} on the road e, and thus becomes the competitor. Note that this model assumes that all agents obey the FIFO policy and C_e^{ik} does not change until the agent i following π_{ik} exits the road e. Using C_e^{ik}, we can express the time-dependent flow on the road e for the agent i's route π_{ik}.

$$f_e^{ik} = \sum_{\pi_{jl} \in C_e^{ik}} p_{jl}. \tag{2}$$

Furthermore, the competitors of the agent $i \in \mathcal{N}$ can be defined as the following set:

$$\mathcal{N}_i = \{j \in \mathcal{N} \setminus \{i\} \mid \exists l \in \mathcal{K}_j,\ \pi_{jl} \in \cup_{k \in \mathcal{K}_i} C_e^{ik}\}.$$

Each agent i collects the route choice probabilities $\boldsymbol{p}_j$ of competitors $j \in \mathcal{N}_i$ through communication networks.

3.3 Distributed Route Selection under Consideration of Time-Dependent Road Usage

In this section, we show how the distributed route selection scheme calculates the optimal route choice probability under the consideration of the time dependency among agents.

The cost of route π_{ik} for agent i can be expressed by the sum of cost of each road along route π_{ik}.

$$c_{ik} = \sum_{\forall e \in \pi_{ik}} c_e(f_e^{ik}),$$

where $c_e(\cdot)$ denotes the cost of the road e under the flow f_e^{ik}, which is a non-decreasing function. From Eqs. (1) and (2), we should note here that $c_e(\cdot)$ depends on $\widetilde{t}_e$. This is a kind of the chicken or egg situations, and thus it is hard to obtain accurate $\widetilde{t}_e$ for each road $e \in \mathcal{E}$ before calculating the path cost c_{ik} $(i \in \mathcal{N}, k \in \mathcal{K}_i)$. In this paper, we simply regard $\widetilde{t}_e$ as the lower bound of travel time on the road e, $\underline{t}_e$, which is the travel time without any congestion on the road e. In Section 4.1.2, we will show this simple assumption contributes to the congestion alleviation but we also plan to apply more sophisticated estimation methods [18], [19].

We assume that each agent $i \in \mathcal{N}$ measures the goodness of the current route choice probabilities $\boldsymbol{p}_i$ based on the local cost V_i [12]:

$$V_i = \sum_{k \in \mathcal{K}_i} p_{ik} c_{ik} - c_{ik_i^*} = \sum_{k \in \mathcal{K}_i} p_{ik}(c_{ik} - c_{ik_i^*}), \tag{3}$$

where $k_i^* = \arg\min_{k \in \mathcal{K}_i} c_{ik}$. Eq. (3) means the difference between the expected path cost under $\boldsymbol{p}_i$ and the minimum path cost. It is rational for each agent i to aim at adjusting the route choice probabilities $\boldsymbol{p}_i$ such that V_i approaches to 0. $V_i = 0$ leads to the following two conditions of Wardrop equilibrium [12]:

$$c_{ik} = c_{ik_i^*}, \qquad \text{if } p_{ik} > 0,$$
$$c_{ik} \geq c_{ik_i^*}, \qquad \text{otherwise.}$$

The first condition means that each agent $i \in \mathcal{N}$ selects the minimum-cost path while the second one indicates that unselected paths have equal or larger cost than the minimum cost.

The global cost V is defined as the sum of the local cost V_i of all agents $i \in \mathcal{N}$ [12]:

$$V = \sum_{i \in \mathcal{N}} V_i.$$

When each agent i aims to adjust $\boldsymbol{p}_i$ such that V_i approaches to 0, V also converges to 0. As a result, Wardrop equilibrium is achieved among all agents.

In [12], a distributed gradient controller is proposed, in which each agent $i \in \mathcal{N}$ can control $\boldsymbol{p}_i$ such that $V_i = 0$ in a distributed manner. The distributed gradient controller governs the time derivative of the route choice probabilities using the competitors' current route choice probabilities. We propose the multi-agent distributed gradient controller considering the time dependency of the agents' road usage, which is the extended version of the existing scheme [12].

We can obtain the local cost increase of the route π_{ik}, w_{ik}, by a small change in p_{ik}:

$$w_{ik} = \sum_{j \in \mathcal{N}_i} \frac{\partial V_j}{\partial p_{ik}}. \tag{4}$$

From Eq. (3), the local cost increase of agent j by the small change in p_{ik}, $\partial V_j / \partial p_{ik}$, can be expressed by

$$\frac{\partial V_j}{\partial p_{ik}} = \frac{\partial}{\partial p_{ik}} \sum_{\forall l \in \mathcal{K}_j \setminus \{k_j^*\}} p_{jl}(c_{jl} - c_{jk_j^*})$$
$$= \begin{cases} \displaystyle\sum_{\forall l \in \mathcal{K}_j \setminus \{k_j^*\}} p_{jl} \left(\frac{\partial c_{jl}}{\partial p_{ik}} - \frac{\partial c_{jk_j^*}}{\partial p_{ik}} \right), & \text{if } i \neq j, \\ \displaystyle c_{ik} - c_{ik_i^*} + \sum_{\forall l \in \mathcal{K}_i \setminus \{k_i^*\}} p_{il} \left(\frac{\partial c_{il}}{\partial p_{ik}} - \frac{\partial c_{ik_i^*}}{\partial p_{ik}} \right), & \text{if } i = j. \end{cases}$$

The cost increase of π_{jl} by the small change in p_{ik}, $\partial c_{jl} / \partial p_{ik}$, is expressed by the sum of each edge's cost increase. Since the small change in p_{ik} only affects the cost of edges shared by both π_{ik} and π_{jl}, we can express $\partial c_{jl} / \partial p_{ik}$ as follows:

$$\frac{\partial c_{jl}}{\partial p_{ik}} = \sum_{e \in \pi_{ik} \cap \pi_{jl}} \mathbb{I}(\pi_{ik} \in C_e^{jl}) \frac{\partial c_e(f_e^{jl})}{\partial f}$$
$$- \sum_{e \in \pi_{ik_i^*} \cap \pi_{jl}} \mathbb{I}(\pi_{ik_i^*} \in C_e^{jl}) \frac{\partial c_e(f_e^{jl})}{\partial f}, \tag{5}$$

where $\mathbb{I}(\cdot)$ denotes an indicator function. The right-hand side of the equation denotes the sum of the cost derivative in the edge level when the corresponding route is included in the set of time-dependent competitive routes of the road e for the agent j following the route π_{jl}. Note that $p_{ik_i^*}$ may also change depending on the small change in p_{ik}.

As in [12], we can finally obtain the following distributed gradient controller per unit time of τ $(\tau > 0)$:

$$\frac{d\boldsymbol{p}_i}{d\tau} = -\gamma V_i \frac{\boldsymbol{y}_i - (\mathbf{1}_{K_i}^\top \boldsymbol{y}_i)\boldsymbol{e}_{k_i^*}}{||\boldsymbol{y}_i||^2}. \quad (6)$$

γ ($\gamma > 0$) denotes a learning rate. $\mathbf{1}_{K_i}$ denotes a column vector of size K_i with all elements set to be 1. $\boldsymbol{e}_{k_i^*}$ denotes a column vector of size K_i, where k_i^*th element is set to be 1 and the remaining elements are set to be 0. $\boldsymbol{y}_i = (y_{i1}, \ldots, y_{iK_i})^\top$ is defined as follows for $k \neq k_i^*$:

$$y_{ik} = \begin{cases} 0 & \text{if } p_{ik} = 0, w_{ik} > 0 \text{ or } p_{ik} = 1, w_{ik} < 0, \\ w_{ik} & \text{otherwise.} \end{cases} \quad (7)$$

When each agent $i \in \mathcal{N}$ adjusts $\boldsymbol{p}_i$ according to Eq. (6), the global cost V reaches to zero, and thus the Wardrop equilibrium is achieved.

3.4 Convergence Analysis

In this section, we show that the proposed scheme exponentially converges to the steady-state as in the conventional scheme [12]. For simplicity, we assume that $\mathcal{K}_i = \mathcal{K}$ and $K_i = K$.

Theorem 1. *The global cost $V(\tau)$ exponentially decreases at the convergence rate inversely proportional to the product of the number of agents and that of individual route candidates (i.e., NK) under the distributed controller in Eq. (6).*

Proof. The time derivative of V can be expressed by

$$\frac{dV}{d\tau} = \frac{1}{NK}\sum_{i\in\mathcal{N}}\sum_{k\in\mathcal{K}} \frac{\partial V}{\partial p_{ik}} \cdot \frac{\partial p_{ik}}{\partial \tau} = \frac{1}{NK}\sum_{i\in\mathcal{N}} \left(\frac{\partial V}{\partial \boldsymbol{p}_i}\right)^\top \frac{d\boldsymbol{p}_i}{d\tau}.$$

From Eq. (6), this can be rewritten as follows:

$$\frac{dV}{d\tau} = \frac{1}{NK}\sum_{i\in\mathcal{N}} \left(\frac{\partial V}{\partial \boldsymbol{p}_i}\right)^\top \left(-\gamma V_i \frac{\boldsymbol{y}_i - (\mathbf{1}_{K_i}^\top \boldsymbol{y}_i)\boldsymbol{e}_{k_i^*}}{||\boldsymbol{y}_i||^2}\right) = -\frac{\gamma}{NK}\sum_{i\in\mathcal{N}} V_i \left(\sum_{j\in\mathcal{N}} \frac{\partial V_j}{\partial \boldsymbol{p}_i}\right)^\top \left(\frac{\boldsymbol{y}_i - (\mathbf{1}_{K_i}^\top \boldsymbol{y}_i)\boldsymbol{e}_{k_i^*}}{||\boldsymbol{y}_i||^2}\right). \quad (8)$$

The nonzero elements of vector $\boldsymbol{y}_i$ except for k_i^*th element are equal to $\sum_{j\in\mathcal{N}} \partial V_j/\partial \boldsymbol{p}_i$ from Eqs. (4) and (7). In addition, k_i^*th element of $\sum_{j\in\mathcal{N}} \partial V_j/\partial \boldsymbol{p}_i$ becomes 0. Note that we assume the competitive relation among agents, i.e., C_e^{ik} ($i \in \mathcal{N}, k \in \mathcal{K}$), is kept during the calculation. Therefore, Eq. (8) can be rewritten as follows:

$$\frac{dV}{d\tau} = -\frac{\gamma}{NK}\sum_{i\in\mathcal{N}} V_i \left(\frac{\boldsymbol{y}_i^\top \boldsymbol{y}_i}{||\boldsymbol{y}_i||^2}\right) = -\frac{\gamma}{NK} V. \quad (9)$$

Solving the differential equation (9) in terms of τ, we have

$$V(\tau) = V(0) \exp\left(-\frac{\gamma}{NK}\right),$$

where $V(0)$ is the initial global cost. □

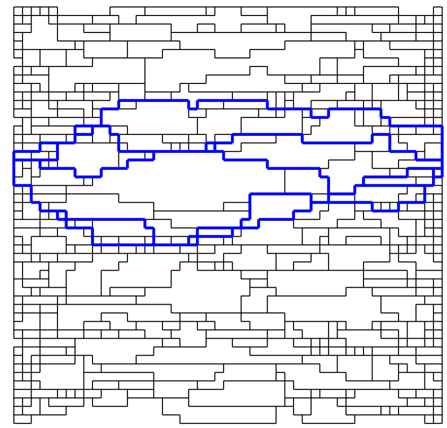


Fig. 2: Route candidates $\boldsymbol{\pi}_{25}$ for agent 25 (blue lines) and route candidates $\{\boldsymbol{\pi}_j\}_{\forall j\in\mathcal{N}}$ for all agents (black lines).

4. Simulation Results

4.1 Evaluation under a Grid-like Network

In this section, we demonstrate the fundamental characteristic of the proposed scheme through simulations using a grid-like road network.

4.1.1 Evaluation Model

To evaluate the fundamental characteristic of the proposed scheme, we use a grid road network consisting of 50×50 nodes (intersections). There are fifty agents ($N = 50$) and each agent $i \in \mathcal{N}$ travels from node $(i, 1)$ to node $(i, 50)$. Note that $(1, 1)$ (resp. $(50, 50)$) is the left-top (resp. right-bottom) node of the grid network. We assume that travel time of each road $e \in \mathcal{E}$ follows the BPR function [25], i.e., $t_e(f_e) = \underline{t}_e(1 + \alpha(f_e/c_e)^\beta)$ where $\underline{t}_e$ denotes the travel time without any congestion on the road e, and c_e denotes the capacity of the road e. α and β represent the degree of the traffic congestion. For each road e, we randomly set $\underline{t}_e$ (resp. c_e) in the range of $[0, 1]$ (resp. $[2, 4]$). We also use $\alpha = 0.15$ and $\beta = 4$.

Each agent $i \in \mathcal{N}$ obtains $K_i = 5$ route candidates $\boldsymbol{\pi}_i$ according to the following procedure. Each agent $i \in \mathcal{N}$ first finds the shortest route from the origin, i.e., node $(i, 1)$, to the destination, i.e., node $(i, 50)$, when the flow of the agent i only exists, i.e., $t_e(1) = \underline{t}_e(1 + \alpha(1/c_e)^\beta)$. Next, it obtains the second route candidate by calculating the shortest route from the origin to the destination under the assumption that the predefined number of road segments, i.e., 30, in the route(s) found so far are unavailable. By repeating this procedure, each agent $i \in \mathcal{N}$ obtains K_i route candidates $\boldsymbol{\pi}_i$, which are exclusive to each other as much as possible. Fig. 2 illustrates an example of route candidates for all agents and those for agent 25 are highlighted by blue color.

As for evaluation criteria, we use the estimated travel

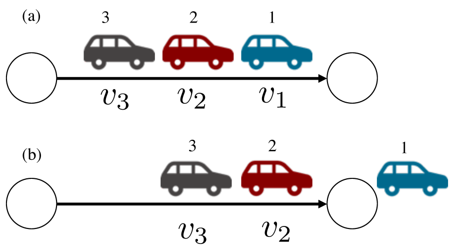


Fig. 3: Congestion model.

time and the actual one. The estimated travel time for agent i, $\widetilde{T}_i$, is defined as the weighted sum of the travel time corresponding each route π_{ik}.

$$\widetilde{T}_i = \sum_{k\in\mathcal{K}_i} p^*_{ik} \sum_{e\in\pi_{ik}} t_e(f^{ik}_e),$$

where the weights are given by $\boldsymbol{p}^*_i$, which is derived by the distributed gradient controller in Section 3. On the other hand, the actual travel time for each agent i, T_i, is the elapsed time between arrival time to a destination and departure time from a origin.

To obtain the actual travel time T_i for each agent $i \in \mathcal{N}$, we implemented a Java simulator. In what follows, we assume that the control interval I_M is sufficiently large and the route calculation is conducted once at the beginning of the simulation. Each agent $i \in \mathcal{N}$ first calculates the route choice probabilities $\boldsymbol{p}^*_i$, and then selects one of the candidates, k^*_i, according to $\boldsymbol{p}^*_i$. We define the deterministic version of $\boldsymbol{p}^*_i$ as $\widehat{\boldsymbol{p}}_i = (\widehat{p}_{i1}, \ldots, \widehat{p}_{iK_i})^\mathsf{T}$. $\widehat{\boldsymbol{p}}_i$ is a vector of size K_i where k^*_ith element is set to be 1 and the remaining elements are set to be 0.

Fig. 3 shows the congestion model used in the simulator. There are three agents traveling in the same direction on the road e. Suppose that the agent i ($i = 1, 2, 3$) selects the route π_{i1}. From Eq. (2), we obtain $f^{11}_e = \widehat{p}_{11} = 1$, $f^{21}_e = \widehat{p}_{11} + \widehat{p}_{21} = 2$, and $f^{31}_e = \widehat{p}_{11} + \widehat{p}_{21} + \widehat{p}_{31} = 3$, in Fig. 3a. As a result, the agent i moves as the speed of $v_i = d_e/t_e(f^{i1}_e)$ where d_e is the length of the road e. When the agent 1 exists on the road e (Fig. 3b), the time-dependent flow for the agents 2 (resp. 3) is updated to $f^{21}_e = \widehat{p}_{21} = 1$ (resp. $f^{31}_e = \widehat{p}_{21} + \widehat{p}_{31} = 2$), and thus the corresponding speed also changes.

For comparison purpose, we also use the conventional scheme, which is the distributed route selection scheme based on the classical congestion game [12], i.e., $C^{ik}_e = C_e$. In what follows, we show the average of 100 independent experiments.

4.1.2 Average and Maximum Travel Time among Agents

Table 2 shows the estimated travel time $\widetilde{T}_i$ and the actual one T_i for both schemes in terms of the average and maximum values. We first focus on the difference between the estimated travel time and actual one for each scheme. We observe that the proposed scheme can more accurately estimate the travel time than the conventional scheme. In particular, the relative estimation error, i.e., $(\widetilde{T}_i - T_i)/T_i$, of the proposed scheme is 0.036 (resp. 0.037) in case of the average (resp. maximum) travel time, which is much smaller than that of the conventional scheme (i.e., 0.19 (resp. 0.15) in case of the average (resp. maximum) travel time).

Table 2: Comparison between estimated travel time and actual one (grid-like network case).

Scheme	$\widetilde{T}_i$ [min]			T_i [min]		
	avg.	max.	std.	avg.	max.	std.
Proposed scheme	23.1	25.1	1.32	22.3	24.2	1.05
Conventional scheme [12]	28.0	29.8	0.87	23.5	25.9	1.17

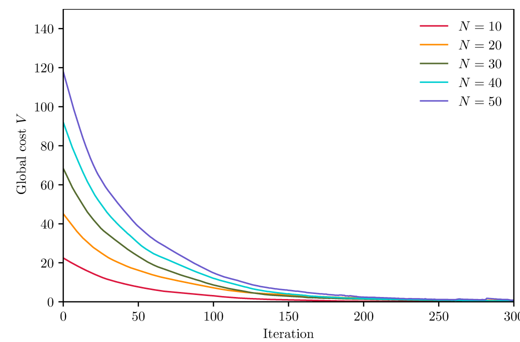


Fig. 4: Impact of N on the convergence property ($K = 5$).

Next, we focus on the performance difference between the proposed scheme and the conventional scheme. We observe that the proposed scheme can improve the average (resp. maximum) actual travel time by 5.1% (resp. 6.6%) compared with the conventional scheme. The static flow assumption used in the conventional scheme considers the worst congestion case while the time-dependent flow assumption in the proposed scheme seems to succeed in estimating more possible congestion level of each road.

4.1.3 Convergence Property

Finally, we evaluate the convergence property of the proposed scheme. Fig. 4 shows the transition of the global cost V when $K = 5$ and N is set to be $10, 20, 30, 40$, and 50. In addition, Fig. 5 also depicts the transition of V when $N = 50$ and K is set to be $2, 3, 4$, and 5. In these figures, we first observe that the global cost V exponentially decreases with the number of iteration. We also observe that the convergence rate is inversely proportional to both the number of agents, N, and that of individual route candidates, K.

4.2 Evaluation under a Real Road Network

In this section, we evaluate the practicality of the proposed scheme through simulations using a real road network, i.e., the central part of Nagoya city, Japan.

4.2.1 Evaluation Model

We use the digital road map of 4.7 [km] × 4.5 [km] east

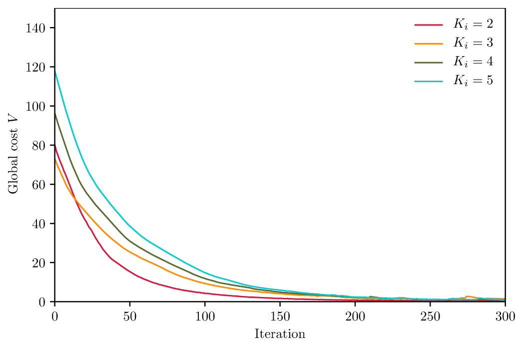


Fig. 5: Impact of K on the convergence property ($N = 50$).

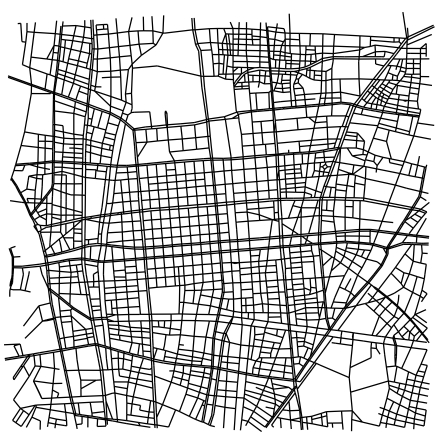


Fig. 6: 4.7 [km] × 4.5 [km] east area of Nagoya station in Japan [26].

area of Nagoya station in Japan, which is provided by Japan Digital Road Map Association [26]. The internal graph structure is composed of 3,173 vertices and 5,013 edges (Fig. 6). Each road on the map has attribute information, i.e., road length, the number of lanes, and speed limit, which can be used for parameters, i.e., $\underline{t}_e$ and c_e, of the BPR function. We set $\underline{t}_e$ by considering the road length and the speed limit, and c_e based on the number of lanes. As for other parameters, we use the same settings in Section 4.1.1, i.e, $\alpha = 0.15$ and $\beta = 4$.

For more realistic evaluation, we use the ordinary flow of people in the target area called people flow data [27]. The people flow data involve the number of people in the target road network and each person's origin and destination with its transportation method at a certain interval, e.g., an hour. We focus on the start of office hour, i.e., 8:00-8:59, where 1,197 vehicles exist in the road network. In what follows, we show the average of 10 independent simulations.

4.2.2 Average and Maximum Travel Time among Agents

Table 3 presents the estimated travel time $\tilde{T}_i$ and the actual one T_i for both schemes in terms of average and maximum values. We first focus on the difference between the estimated travel time and actual one for each scheme. We observe that the proposed scheme exhibits more accurate estimation than

Table 3: Comparison between estimated travel time and actual one (realistic network case).

Scheme	$\tilde{T}_i$ [min]			T_i [min]		
	avg.	max.	std.	avg.	max.	std.
Proposed scheme	2.66	7.71	1.54	2.65	7.70	1.53
Conventional scheme [12]	3.49	41.2	4.04	2.72	8.07	1.60

the conventional scheme. Specifically, the relative estimation error of the proposed scheme is 0.004 (resp. 0.001) in case of average (resp. maximum) travel time, which is much smaller than that of the conventional scheme, i.e., 0.28 (resp. 4.11) in case of average (resp. maximum) travel time.

Next, we focus on the performance difference between the proposed scheme and conventional scheme. We confirm that the proposed scheme can improve the average (resp. maximum) actual travel time by 2.5% (resp. 4.6%) compared with the conventional scheme.

5. Conclusion

In the classical routing game, all the roads composing the route are assumed to be used simultaneously and continuously. However, this assumption should be reconsidered since the congestion level would change over time. In this paper, we have proposed a multi-agent distributed route selection scheme based on a gradient descent method considering time dependency among agents' road usage. In the proposed scheme, each agents calculates the route choice probabilities by using the estimated time-dependent flow on each road in the distributed manner. We have first proved that the proposed scheme exponentially converges to the steady-state at the convergence rate inversely proportional to the product of the number of agents and that of individual route candidates. Through the simulation results under the grid-like road network, we have shown that the proposed scheme can improve the actual travel time by 5.1%, compared with the conventional scheme. Furthermore, we have also evaluated the practicality of the proposed scheme through simulations under the realistic road network of Nagoya city. We have confirmed that the proposed scheme can effectively estimate traffic load and improve the actual travel time by 2.5% compared with the conventional scheme.

In future work, we plan to investigate how the proposed scheme can improve the actual travel time by controlling the interval I_{M}, which will contribute to increase the estimation accuracy at the expense of the computational complexity. In addition, we also consider to apply the existing predictive traffic congestion models to estimate the time-dependent flow more accurately. Combination of the proposed scheme with the selfish yet optimal routing [28] is also a possible future direction.

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Takanori HARA received the M. Eng. degree from Nara Institute of Science and Technology, Japan, in 2018. He is currently working towards the Ph.D degree with Nara Institute of Science and Technology, Japan. He is also a Research Fellow of the Japan Society for the Promotion of Science (DC2). His research interests include evacuation guiding, route planning, NFV network, and game-theoretic approach.

Masahiro SASABE received the B.S., M.E., and Ph.D. degrees from Osaka University, Japan, in 2001, 2003, and 2006, respectively. He was an Assistant Professor with the Cybermedia Center, Osaka University from 2004 to 2007, an Assistant Professor of Graduate School of Engineering, Osaka University from 2007 to 2014. He is currently an Associate Professor of Graduate School of Science and Technology, Nara Institute of Science and Technology, Japan. His research interests include P2P/NFV networking, game-theoretic approaches, and network optimization. Dr. Sasabe is a member of IEEE and IEICE.

Shoji KASAHARA received the B. Eng., M. Eng., and Dr. Eng. degrees from Kyoto University, Kyoto, Japan, in 1989, 1991, and 1996, respectively. Currently, he is a Professor of Graduate School of Information Science, Nara Institute of Science and Technology, Nara, Japan. His research interests include stochastic modeling and analytics of large-scale complex systems based on computer/communication networks.