Perfect Cell Partitioning in Micro-Cellular Networks

Akiko Miyagawa, Masahiro Sasabe, *Member, IEEE*, and Hirotaka Nakano, *Member, IEEE*

***Abstract*— In recent years, new mobile communication services, such as downloading movies and distributing various shop information with sound and animation, have appeared. These services require much more bandwidth to smoothly transmit multimedia data to users. Therefore, transmission rate becomes more important. A cellular system is an infrastructure used in a mobile communication between a cellular phone, called a node, and a base station. In a cellular system, a base station can smoothly communicate with nodes in the cell by avoiding overlap of frequency range with its adjacent cells. From the viewpoint of graph theory, however, that needs to divide the original frequency range into at least four sub-ranges. If a base station avoids overlapping its communication area, called a cell, with its adjacent base stations by appropriately controlling its transmission power, that is perfect cell partitioning, it can use the whole of the original frequency range. In this paper, we investigate the feasible area of the perfect cell partitioning. We first make modeling the perfect cell partitioning. Then, we provide geometrical analysis to clarify the feasible area of the perfect cell partitioning. Through several simulation experiments, we verify the accuracy of the analysis and show the effectiveness of our proposal. Simulation results show that the perfect cell partitioning is effective when the number of nodes in a cell is lower than 6 and the occupation ratio is 1.2.**

***Index Terms*— perfect cell partitioning, cellular system, mobile communication, radio interference, transmission power control**

I. Introduction

A mobile communication technology which plays an important role in a ubiquitous network has attracted extensive research efforts in recent years. In the traditional mobile communication service, voice and mail data occupied the large portion of traffic. Such data can be transmitted at relatively low bit rate. In recent years, cellular phone service providers offer a flat-rate plan independently of the amount of consumed traffic. This new type of service plan enhances users to download short movies and music. In addition, a shop may want to distribute advertising information to people when they get close to it. In both types of service, network capacity and transmission rate become more important.

A cellular system [1][2] is an infrastructure used in a mobile communication between a cellular phone, called a node, and a base station. In the system, a service region is divided into multiple subregions, called ”cells.” In every cell, a base station is located at the center and communicates with a node using a wireless connection. A base station can smoothly communicate with nodes in the cell by avoiding overlap of frequency range with its adjacent cells. From the viewpoint of graph theory, however, that needs to divide the original frequency range into at least four sub-ranges [3][4]. This leads to deteriorate the transmission rate. If a base station avoids overlapping its cell with its adjacent base stations by appropriately controlling its transmission power, that is perfect cell partitioning, it can use the whole of the original frequency range. As a result, the perfect cell partitioning achieves about four times higher transmission rate than the traditional cellular system.

In this paper, we investigate the feasible area of the perfect cell partitioning on the assumption that a telecom service provider offers a high-speed data transfer service to users. A telecom service provider is responsible for giving a chance for communication to all nodes in its service area. In such a situation, the perfect cell partitioning is accomplished only when a base station can adjust the size of the cell so that there is no node in the vicinity of border among adjacent cells. In such a situation, nodes in a cell satisfying the condition can enjoy the high-speed information distribution service.

We expect that the perfect cell partitioning tends to success when the cell size becomes small. This is because nodes in the vicinity of the border among cells decrease in response to the reduction of the cell size. In recent years, various wireless technologies whose cell size is relatively small [1], such as Bluetooth [5] and ZigBee [6], have been widely deployed. Thus, the perfect cell partitioning is expected to be one of key concepts achieved over these technologies.

We first make modeling the above type of perfect cell partitioning based on a radio interference model. Then, by using analytical approach, we derive the success probability of perfect cell partitioning under the following assumptions.

- The region is divided into multiple cells whose shape is a regular hexagon.
- Nodes are located at random positions in the region.
- Direction of communication is one way from a base station to a node. Information distribution services are categorized to this type of communication.

Through simulation experiments, we verify the analysis and show the effectiveness of the perfect cell partitioning.

The remainder of this paper is organaized as follows. In Section 2, we introduce the model for perfect cell partitioning. In Section 3, we derive the success probability of perfect cell partitioning using geometrically analytical approach. We conduct several simulation experiments in Section 4. Finally, Section 5 gives conclusions of this paper.

II. Perfect Cell Partitioning

In this section, we explain the condition that the perfect cell partitioning successes. We first introduce a model of radio interference in wireless networks. In general, a radio wave is attenuated in inverse proportion to αth power of distance [7][8]. Suppose that base station bs emits a radio wave with transmission power P. Then, power $P(i)$ that node X_i receives from bs is expressed as

$$P(i) = \frac{P}{|X_i - bs|^\alpha}. \quad (1)$$

In other words, perceived radio quality at X_i is differentiated by distance r from bs. Therefore, we define the three zones around bs as follows.

- If $|X_i - bs| \leq r$, X_i is in a success zone where it can receive data from bs correctly.
- If $r < |X_i - bs| \leq \Delta r$, X_i in a noise zone where it receives data from bs as noise.
- If $\Delta r < |X_i - bs|$, X_i is in no interference zone where it does not receive data from bs.

Here, we call Δ as occupation ratio that determines the area occupied by bs.

Then, we model the perfect cell partitioning using this radio interference model. First, we assume that a base station is responsible for the connections to all nodes in a range of its maximum transmission power. The condition where bs can connect to all of them is as follows.

$$|X_k - bs| \geq \Delta |X_{far} - bs|, \quad (2)$$

where X_{far} is the most distance node from bs in a range of its maximum transmission power and X_k is every other node out of the range. Equation (2) indicates that bs can achieve the perfect cell partitioning only when every node belonging other base stations is in the no interference zone.

To realize this mechanism, each base station has to collect information on nodes in the range of maximum transmission power. In addition, it also needs to know every other node belonging its neighboring base stations. This information can be obtained by exchanging node lists among neighboring base stations. The detail of the mechanism is a future work.

III. Geometric analysis

In this section, we analyze the success probability of perfect cell partitioning.

First, suppose that N_r adjacent cells are periodically located in the region. If N_r increases, accuracy of the analysis grows up. Figure 1 displays a sample distribution of nodes and base stations when $N_r = 4$. A circle with dotted line and that with solid line express a region and a cell, respectively. Each cell that consists of the recurring unit is shaded. A triangle and a dot represent a base station and a node, respectively.

As mentioned in the previous section, we assume a cell is shaped as a regular hexagon. Suppose that the length of each edge of the hexagon is R. The radius R_{eq} of the equivalent circle whose area is equal to that of the hexagon becomes $\sqrt{\frac{3\sqrt{3}}{2\pi}}R$. Note that we do not lose generality even if we set R_{eq} to 1. In the following analysis, we set R_{eq} to 1. Moreover, suppose that node density in a cell is n and the occupation ratio is Δ. We also define the success probability of perfect cell partitioning $P_{success}(n)$ as the probability that all nodes in a cell can communicate with its base station.

In what follows, we analytically derive $P_{success}(n)$ when $N_r = 1$, 4, and 7.

A. In the case of $N_r = 1$

When $N_r = 1$, only one type of cell is lined with the region. We focus on the neighboring seven cells in the region, as illustrated in Fig. 2. A hexagon and a solid line circle in Fig. 2 express a cell and a range of max transmission power of a base station, respectively. Define the center cell in the seven adjacent cells as a main cell and the others as neighboring cells. In what follows, we focus on the main cell. Suppose that r is the distance between the base station bs and the most distant node from bs in the main cell. Since the position of a node is invariant among cells when $N_r = 1$, the dinstance between a base station and the most distant node from it equals to r in every cell. For successful communication in the main cell, the nodes in the adjacent cells cannot exist in the noise zone of the main cell. As a result, the most distant node from bs can exist on the thick lines in Fig. 2. Since this is satisfied for every $0 \leq r \leq 1$, the area that the most distant node can exists is expressed as

$$S_1 = \int_0^1 S_b(r)dr, \quad (3)$$

where $S_b(r)$ is the sum of the length of the thick lines.

The success probability equals to the probability that all n nodes in a cell exists in the region explained above. As a result, the success probability per node is

$$P_{success}(n) = \left(\frac{S_1}{\pi R_{eq}^2}\right)^n. \quad (4)$$

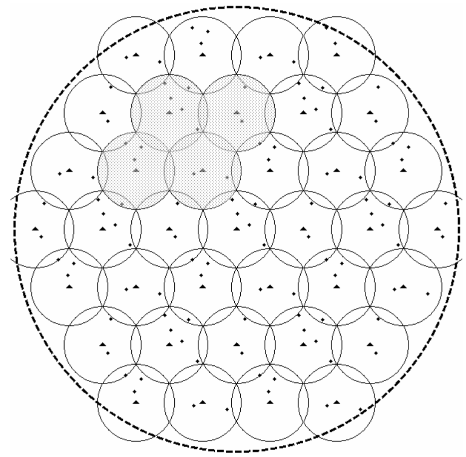

Fig. 1. Sample distribution of nodes and base stations when $N_r = 4$

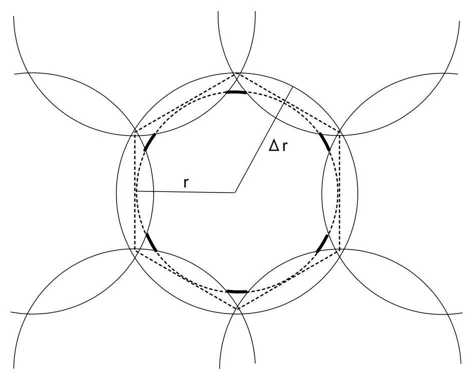

Fig. 2. The region the most distant node from bs can exists when $N_r = 1$

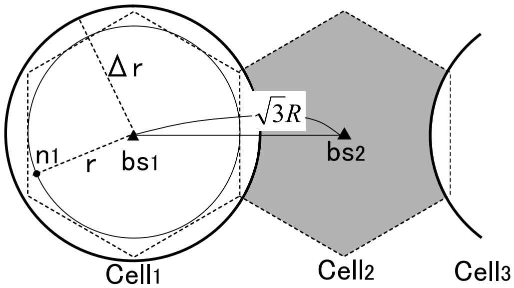

Fig. 3. Effect to the neighboring cell when $N_r = 4$

B. In the case of $N_r = 4$

When $N_r = 4$, four adjacent cells are located in the region as shown in Fig. 1. Define one cell from the four adjacent cells as a main cell and the others as neighboring cells. In what follows, we focus on the main cell. Suppose that there are $k \leq 4n$ nodes in the main cell.

In this case, the success probability of perfect cell partitioning is

$$\begin{aligned} & P_{success}(n) \\ & = \ {}_4\mathrm{C}_1 \left(\frac{1}{4}\right)^{4n} \\ & \quad + \frac{1}{n} \sum_{k=1}^{4n-1} \left\{ k C_4(n,k) P^k_{success}(n,k) \right\}, \end{aligned} \tag{5}$$

where $C_4(n,k)$ is the probability that k nodes are in the main cell and $P^k_{success}(n,k)$ is the probability that all k nodes in the main cell can communicate with their base station. The former factor means the success probability when $k = 4n$, and the latter factor means sum of the success probability when $1 \leq k \leq 4n-1$.

Figure 3 illustrates the main cell, $Cell_1$, and one of the neighboring cells, $Cell_2$, when the distance between the base station bs_1 and the most distant node n_1 from bs_1 is r. In this case, for successful communication in $Cell_1$, the nodes in $Cell_2$ cannot exist in the noise zone of $Cell_1$. Therefore, they must exist in the grey zone in Fig. 3. Suppose that the area of the grey zone is S. The nodes in the other neighboring cells must also exist in the grey zones of their belonging cells. As a result, the probability that $4n-k$ nodes in the three neighboring cells exists in the grey zones becomes

$$p^r_{adj}(n,k,r) = \left(\frac{3S}{3S_{cell}}\right)^{4n-k}, \tag{6}$$

where S_{cell} is the area of a cell. Let $p_{cnt}(n,k,r)$ be the probability that the distance between bs_1 and n_1 becomes r. Consequently, the success probability of perfect cell partitioning for $Cell_1$ when the distance between bs_1 and n_1 is r becomes

$$p^r(n,k,r) = p_{cnt}(n,k,r) p^r_{adj}(n,k,r). \tag{7}$$

Therefore, the probability that all k nodes in $Cell_1$ can communicate with bs_1 is

$$P^k_{success}(n,k) = \int_0^1 k \ p^r(n,k,r) dr. \tag{8}$$

From Eq. (5)∼(9), we can derive $P_{success}(n)$.

C. In the case of $N_r = 7$

When $N_r = 7$, seven adjacent cells are located in the region. Define the cell centered in the seven adjacent cells as a main cell and the others as neighboring cells. In this case, the success probability of perfect cell partitioning in the main cell is

$$\begin{aligned} & P_{success}(n) \\ & = \ {}_7\mathrm{C}_1 \left(\frac{1}{7}\right)^{7n} \\ & \quad + \frac{1}{n} \sum_{k=1}^{7n-1} \left\{ k \ C_7(n,k) P^k_{success}(n,k) \right\}, \end{aligned} \tag{9}$$

where $C_7(n,k)$ is the probability that k nodes are in the main cell and $P^k_{success}(n,k)$ is the probability that all k nodes in the main cell can communicate with its base station. We omit the details of the analysis because the subsequent analysis is similar to that when $N_r = 4$.

IV. Simulation Experiments

In this section, we verify the accuracy of our analysis and show the effectiveness of our proposal. In the following simulations, we set the number of base stations in the region to 250 in order to reduce edge effect. By the edge effect, we cannot precisely simulate our proposal near the boarder of the region as shown in Fig. 1. From results of our pre-examination, the edge effect can be ignored when the number of base stations is more than 200. We vary the number of nodes in the region from 250 to 2500. Thus, the average number of nodes in a cell, that is node density n, ranges from 1 to 10. We evaluate the performance in terms of the success probability that is

$$P_{success} = \frac{N_c}{N_a}, \tag{10}$$

where N_c and N_a are the number of nodes that success to connect a base station and the number of nodes in the region, respectively. In the following results, we show the average of 1000 simulations.

A. Accuracy of analysis

We verify the accuracy of our analysis by comparing simulation results. Figure 4 illustrates how $P_{success}$ varies according to the node density in the case of $\Delta = 1.2$. Since N_r is the number of cells in the recurring unit, increase of N_r contributes to reduction of difference between analytical and simulation results. Furthermore, the difference is at most 12 % when $N_r = 7$. Since the number of cells in the region is 250, this indicates that N_r can be relatively small to improve the accuracy of the analysis.

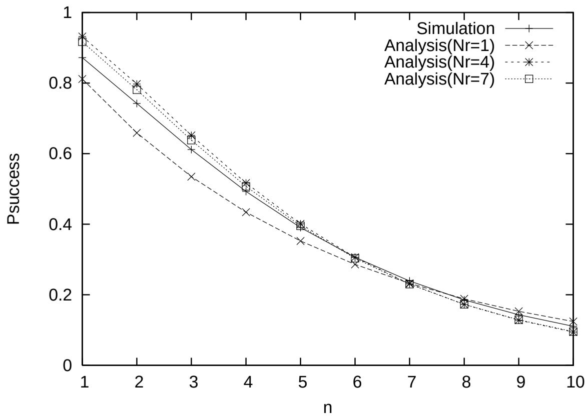

Fig. 4. $P_{success}$ vs. node density ($\Delta = 1.2$)

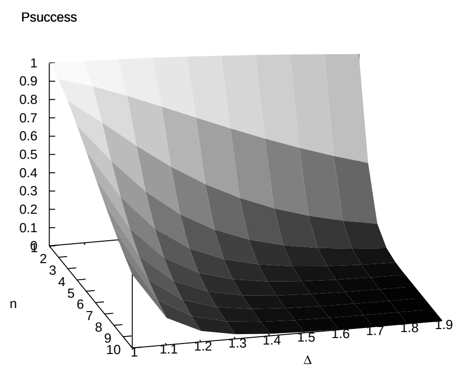

Fig. 5. Transition of $P_{success}$ with node density and Δ

B. Impact of node density and occupation ratio

Figure 5 depicts the transition of $P_{success}$ with node density and Δ. As shown in Fig. 5, $P_{success}$ falls down with the increase of both of node density and Δ. To compare our proposal with the traditional cellular system, we define radio utilization as the product of available frequency range and the success probability. As mentioned before, our proposal can use about four times larger frequency range than the traditional cellular system. Thus, in terms of the radio utilization, our proposal is more effective than the traditional cellular system when $P_{success}$ is larger than 0.25. For example, $P_{success}$ is larger than 0.25 if the node density is not over 6 and Δ is 1.2. This indicates that our proposal is efficient when the node density is low. The deterioration of $P_{success}$ is mainly caused by the randomness of node positions in the region. In an actual situation, a node seems to move around to get a connection to a base station when it cannot communicate. This phenomenon promotes the success of the perfect cell partitioning because nodes in the vicinity of border among cells decrease.

V. Conclusions and future works

In this paper, we proposed the model for perfect cell partitioning to avoid radio interference among adjacent base stations. Then, we clarified the relation between the success probability of perfect cell partitioning and the node density in a cell through both geometrical analysis and simulation experiments.

The simulation results indicate that our proposal is effective when the node density is not over 6. This indicates that our proposal is effective when the node density is low.

As future works, we plan to modify our proposal to adapt to a different service model. In this paper, we try to enhance the utilization of radio resource in a perfect partitioned cell without degrading communication quality in adjacent cells. However, someone may want to improve the success probability of perfect cell partitioning even at the expense of the communication quality of nodes in the noise zone. For example, the following service corresponds to this type of perfect cell partitioning: a shop distributes advertising information from its own base station to nodes getting close to it. In such a case, we expect that the success probability of perfect cell partitioning drastically improves. We hope that perfect cell partitioning becomes one of key technologies to realize the Fourth Generation (4G) Mobile Communication Systems.

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