Shortest Path Tour Problem Based Integer Linear Programming for Service Chaining and Function Placement in NFV Networks

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***Abstract*—Network functions virtualization (NFV) is a new paradigm to achieve flexible and agile network services by decoupling network functions from proprietary hardware and running them on generic hardware as virtual network functions (VNFs). In the NFV network, a network service can be modeled as a sequence of VNFs, called a service chain. Given a connection request (e.g, origin, destination, and a sequence of required functions), we have to solve both the service chaining and function placement problems to find an appropriate service path that optimizes the objective (e.g., minimization of the total path delay) while satisfying the service chain requirements. In this paper, focusing on the similarity between the service chaining problem and the shortest path tour problem (SPTP) and developing the novel network model called augmented network, we formulate SPTP-based integer linear programs (ILPs) for the service chaining and function placement. Through numerical results obtained by the existing solver, we show the SPTP-based ILP for the service chaining can support 1.22–1.85 times as large-scale systems as the existing ILP. Furthermore, we also demonstrate that the ILP for both the service chaining and function placement can shorten the total delay by 15.7% compared with that only for the service chaining.**

***Index Terms*—Network functions virtualization (NFV), service chaining, function placement, integer linear programming (ILP), augmented network, shortest path tour problem (SPTP)**

I. Introduction

NETWORK function virtualization (NFV) can realize flexible and agile network services by decoupling network functions (e.g., routing, firewall, deep packet inspection, and load balancing) from dedicated hardware (e.g., appliances and middleboxes) and running them on generic hardware as virtual network functions (VNFs) [2]. In what follows, the terms VNF and function are used interchangeably. In addition to the flexibility and agility, NFV can also reduce both capital expenditures (CAPEX) and operating expenditures (OPEX) because it can be built on the generic hardware and liberate operators from learning knowledge for the conventional dedicated hardware.

The resource allocation is one of the challenging issues in the NFV networks [3], [4]. We can view a certain network service as a sequence of VNFs, called a service chain (SC) or service function chain (SFC) [5]. Given a connection request with service chain requirements (e.g., an origin node, a destination node, and a sequence of functions), the *service chaining* problem aims to find an appropriate *service path*, which starts from the origin node and ends with the destination node while executing the corresponding VNFs at the intermediate nodes in the required order. Furthermore, the locations of functions in the NFV network also affect the effectiveness of the service path, and thus the *function placement* problem also arises [6].

Several studies noticed the similarity between the shortest path tour problem (SPTP) [7]–[9] and the service chaining problem [10], [11]. The SPTP is a variant of the shortest path problem (SPP) and tries to find a shortest path from an origin node to a destination node such that the path must visit a sequence of disjoint node subsets $\mathcal{T}_1, \ldots, \mathcal{T}_K$ in this order. For example, $\mathcal{T}_k$ $(k = 1, \ldots, K)$ can be regarded as touristic spots on a certain tour. Bhat and Rouskas first showed the possibility of modeling the service chaining problem as the SPTP [10]. Gao and Rouskas proposed the SPTP-based heuristic algorithm for the online service chaining [11].

Recently, Andrade and Saraiva proposed an integer linear program (ILP) for the constrained SPTP [12]. The constrained SPTP is a special case of the SPTP such that the path does not include repeated edges. In our previous work [1], combining this approach and a novel network model called *augmented network*, we showed that the service chaining problem can exactly be modeled as an SPTP-based ILP, which minimizes the total delay of the service path. Although there are various types of ILPs for the service chaining [13]–[18], to the best of our knowledge, this was the first work that exactly formulated the service chaining problem as the SPTP-based ILP. In this paper, we further extend this ILP for both the service chaining and function placement, which can determine the appropriate service path as well as the appropriate number and locations of VNFs in the NFV networks. Through numerical results obtained by solving the proposed ILPs using the existing solver CPLEX 12.8 [19], we evaluate the proposed ILPs in terms of the scalability and effectiveness of the resource allocation.

The main contributions of this paper are as follows:

1) We reveal the exact relationship between the conventional SPTP and the NFV-related problems (i.e., service chaining and function placement) by developing two kinds of SPTP-based ILPs, $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ and $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$, with the help of the combination of the ILP formulation of the conventional SPTP and the augmented network. $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ is the SPTP-based ILP for the service chaining

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while $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$ is that for both the service chaining and function placement.

2) We propose a new type of NFV network model called augmented network, which can connect the NFV-related problems to the conventional SPTP. The augmented network can be much smaller than the existing NFV network models (i.e., layered graph and expanded network) especially when the number of functions required in the service chain increases.
3) Through numerical results obtained by solving the proposed ILPs using the existing solver CPLEX, we will show $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ can support 1.22–1.85 as times large-scale systems as the existing ILP over the expanded network. More specifically, $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ (resp. $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$) can support 473 and 1556 (resp. 441 and 1464) physical nodes within 1 and 10 execution time [s], respectively. We further demonstrate that $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$ can reduce the total delay of the service path by 15.7% and the average physical node (resp. link) utilization by 26.8% (resp. 60.9%) compared with $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$.

The rest of the manuscript is organized as follows. Section II gives the related work. After introducing the system model in Section III, we develop the two kinds of SPTP-based ILPs for the service chaining and function placement in NFV networks. We demonstrate the scalability and effectiveness of the proposed SPTP-ILPs through numerical results in Section V. Finally, Section VI gives the conclusions and future work.

II. Related Work

The resource allocation is one of the most challenging problems in NFV networks because it requires to deal with various types of requirements (e.g., service (function) chaining, function placement, scheduling, and demand provisioning) as well as their complex dependence. The recent survey on NFV networks can be found in [3], [4], [6]. In what follows, we introduce existing studies on the ILP formulation for the NFV-related problems (i.e., service chaining and function placement) and SPTP-related work.

Service chaining is one of the major NFV resource allocation problems, which aims to find an appropriate service path under the constraints of function locations and service chain requirements (e.g., processing/bandwidth demand and a sequence of functions). It is a kind of combinatorial optimization problems and there are multiple studies on the mathematical formulation of the service chaining problem [13]–[18]. Nguyen et al. formulated the service chaining problem as an ILP over an expanded network to minimize the blocking probability of service chain requests [13]. Huin et al. developed an ILP over a layered graph to minimize the total bandwidth usage [14]. Gupta et al. also formulated an ILP to minimize the total bandwidth usage [15]. Savi et al. modeled an ILP to minimize the number of active nodes by considering the processing-related costs, i.e., context switching costs and upscaling costs, in a VNF consolidation scenario where multiple VNFs were served at the same hardware [16]. Nejad et al. [17] aimed to maximize the total revenue. Hyodo et al. formulated an ILP to minimize the placement cost and bandwidth usage under the relaxation of VNF order and non-loop constraints [18].

Path selection-based service chaining aims to find an appropriate service path from given path candidates. D'Oro et al. applied the non-cooperative game theory to achieve service chaining a distributed manner [20]. They also proposed a two-stage Stackelberg game composed of leading servers and following users to achieve the distributed resource allocation and orchestration [21].

Function placement also plays an important role to minimize the total path delay as well as utilize the limited resource effectively. Bhamare et al. formulated an ILP for the VNF placement across geographically distributed clouds to minimize the total response time to the end-users in the network [22]. Li et al. formulated a facility location problem based ILP for the VNF placement to minimize the resource consumption [23]. Tomassilli et al. formulated the VNF placement under the SFC constraints as a set cover problem and proposed approximation algorithms to solve it [24].

There are also several studies that aim to solve both the service chaining and function placement. Sallam et al. first proposed a transformation of the network to calculate SFC-constrained shortest path candidates, then proposed two kinds of ILPs for SFC-constrained maximum flow and VNF placement [25]. Gouareb et al. modeled an ILP for both the VNF routing and placement across the physical nodes to minimize the edge-cloud latency [26]. Soualah et al. proposed an ILP for both the service chaining and function placement to minimize the resource usage when VNFs are shared across tenants [27].

Recently, some researchers noticed that the service chaining is similar to the conventional SPTP [10], [11], [28]. The SPTP is a variant of the SPP where some intermediate nodes (locations) should to be visited in a predefined order. Festa et al. achieved polynomial-time reduction of the SPTP to a classical SPP over a modified digraph and proposed heuristics to solve the SPTP [9]. Ferone et al. studied constrained SPTP, which is a variant of the SPTP without repeated links, proved that it belonged to the complexity class of NP-complete, and proposed a branch-and-bound based heuristic [29]. Ferone et al. further proposed a mathematical model and new branch-and-bound heuristics for the CSPTP [30]. Recently, Andrade and Saraiva developed an ILP for the constrained SPTP [12]. They also proposed a Lagrangian-based heuristic framework to solve the constrained SPTP [31]

As for the application of the SPTP in the NFV networks, Bhat and Rouskas showed that service chaining could be modeled as SPTP [10]. They proposed a heuristic algorithm to solve the SPTP and compared it with existing algorithms. Gao et al. proposed a SPTP-based online path-selection algorithm to minimize the maximum network congestion [11]. Focusing on the similarity between the service chaining and SPTP, Liu et al. tackled the scaling problem of service chaining with the help of the combination of the multistage graph and the max-flow min-cut theory [28].

To the best of our knowledge, this is the first work that exactly formulates SPTP-based ILPs for the NFV-related problems (i.e., service chaining and function placement).

TABLE I
NOTATIONS IN THE MODEL.

Notation	Definition
G	Physical network $G = (\mathcal{V}, \mathcal{E})$
$\mathcal{V}$	Set of physical nodes, $V = \lvert\mathcal{V}\rvert$
$\mathcal{E}$	Set of physical links, $E = \lvert\mathcal{E}\rvert$
$\mathcal{F}$	Set of functions, $\mathcal{F} = \{f_1, \ldots, f_F\}$, $F = \lvert\mathcal{F}\rvert$
$\mathcal{F}_i$	Set of functions contained in physical node $i \in \mathcal{V}$
N_f	Number of physical nodes capable of function f
c	Connection with origin o_c and destination d_c
$\mathcal{R}_c$	Sequence of functions $(f_{c,1}, \ldots, f_{c,K_c})$ required by c
$\mathcal{K}_c, \mathcal{K}_c^+$	$\mathcal{K}_c = \{1, \ldots, K_c\}, \mathcal{K}_c^+ = \{1, \ldots, K_c + 1\}$
$\widehat{\mathcal{V}}$	Set of imaginary nodes, $\widehat{\mathcal{V}} = \{\hat{v}_f\}_{f \in \mathcal{F}}$, where imaginary node $\hat{v}_f$ is responsible for function f
$\widehat{\mathcal{E}}^{\text{in}}$	Set of links incoming to imaginary nodes, $\widehat{\mathcal{E}}^{\text{in}} = \{(v, \hat{v}) \mid v \in \mathcal{V}, \hat{v}_f \in \widehat{\mathcal{V}}, f \in \mathcal{F}_v\}$
$\widehat{\mathcal{E}}^{\text{out}}$	Set of links outgoing from imaginary nodes, $\widehat{\mathcal{E}}^{\text{out}} = \{(\hat{v}, v) \mid \hat{v}_f \in \widehat{\mathcal{V}}, v \in \mathcal{V}, f \in \mathcal{F}_v\}$
G^+	Augmented network of G, $G^+ = (\mathcal{V}^+, \mathcal{E}^+)$
$\mathcal{V}^+$	Set of nodes in G^+, $\mathcal{V}^+ = \mathcal{V} \cup \widehat{\mathcal{V}}$, $V^+ = \lvert\mathcal{V}^+\rvert$
$\mathcal{V}_i^+$	Set of node i's neighbors in G^+
$\mathcal{E}^+$	Set of links in G^+, $\mathcal{E}^+ = \mathcal{E} \cup \widehat{\mathcal{E}}^{\text{in}} \cup \widehat{\mathcal{E}}^{\text{out}}$
$\mathcal{S}_c$	Service path for c, $(\mathcal{S}_{c,1}, \ldots, \mathcal{S}_{c,K_c+1})$
$\mathcal{S}_{c,k}$	kth sub service path with origin $a_{c,k}$ and destination $b_{c,k}$
b_c	Bandwidth requirement of c
p_c^{node}	Processing requirement of c for traversing a node
$p_{c,f_{c,k}}^{\text{func}}$	Processing requirement of $f_{c,k} \in \mathcal{R}_c$ at a node
$B_{i,j}$	Residual bandwidth of link (i,j) at arrival of c
P_i	Residual processing capacity of node i at arrival of c
$d_{i,j}^{\text{link}}$	Propagation delay of link $(i,j) \in \mathcal{E}$
d_i^{node}	Traversal delay of node $i \in \mathcal{V}$
$d_{\hat{v}_f,v}^{\text{func}}$	Processing delay of function f of node $v \in \mathcal{V}$
$x_{i,j}^{c,k}$	Binary decision variables: 1: if link (i,j) is included in $\mathcal{S}_{c,k}$, 0: otherwise

III. SYSTEM MODEL

In this section, we describe the system model considered in this manuscript, from the viewpoint of service chain request, augmented network, and service path. Table I summarizes the notations used in this paper. In Section IV, we will provide two kinds of ILPs: one is only for the service chaining, $\text{ILP}_{\text{SC}}^{\text{SPTP}}$, and another is for both the service chaining and function placement, $\text{ILP}_{\text{SCFP}}^{\text{SPTP}}$. In this section, for simplicity in explanation, we mainly focus on the service chaining and give the information related to both the service chaining and function placement if required.

A. Service Chain Request

We consider an NFV network where connection requests randomly arrive as in [13]. The orchestrator waits for C ($C = 1, \ldots, C_{\max}$) requests and solves the service chaining (and function placement) problem for the collected requests $\mathcal{C}$. The way of processing the request(s) can be categorized into three types, i.e., online, batch, and offline, depending on the size of $\mathcal{C}$, i.e., C. In case of the online processing ($C = 1$), the orchestrator serves the connection request just after its arrival. If the orchestrator knows all the requests ($C = C_{\max}$) in advance, it solves the service chaining problem for them at once, and thus this results in the offline processing. In other cases, i.e., $C = 2, \ldots, C_{\max} - 1$, the orchestrator adopts the batch processing with size of C.

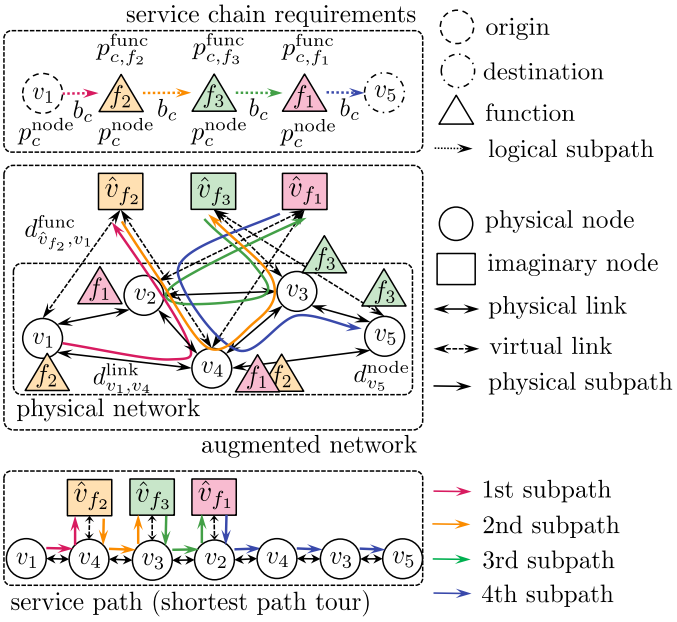


Fig. 1. Overview of service chaining: relationship between service chain requirements, augmented network, and service path ($C = 1$).

A connection $c \in \mathcal{C}$ has service chain requirements, $(o_c, d_c, \mathcal{R}_c, b_c, p_c^{\text{node}}, \{p_{c,f_{c,k}}^{\text{func}}\}_{k=1,\ldots,K_c})$. o_c (resp. d_c) is a physical node that the connection c starts from (resp. ends with). $\mathcal{R}_c$ is a sequence (an ordered set) of K_c ($K_c > 0$) functions $(f_{c,1}, \ldots, f_{c,K_c})$ with which the connection c will be served in this order. In general, the sequence of functions $\mathcal{R}_c$ can be more complex processing order rather than the sequential order, e.g., inclusion of split and merge. In this paper, we mainly focus on the sequential order, which results in the simple yet novel ILP of the service chaining problem. As for the bandwidth/processing demand, we use the same assumptions as in [13]. We consider that b_c ($b_c > 0$) is a fixed value, e.g., constant bit rate (CBR). Whenever the connection c passes through a physical node, it needs a processing capacity p_c^{node} ($p_c^{\text{node}} > 0$) to process the packets through the node. In addition, the connection c requires a processing capacity $p_{c,f_{c,k}}^{\text{func}}$ ($p_{c,f_{c,k}}^{\text{func}} > 0$) for executing each kth function $f_{c,k}$ in $\mathcal{R}_c$. An example of the service chain requirements for one connection c is shown in the top layer of Fig. 1 where $(o_c, d_c) = (v_1, v_5)$, $\mathcal{R}_c = (f_{c,1}, f_{c,2}, f_{c,3}) = (f_2, f_3, f_1)$, and bandwidth/processing requirements are given as b_c, p_c^{node}, and $\{p_{c,f}^{\text{func}}\}_{f \in \mathcal{R}_c}$, respectively.

Note that our model can also aggregate the same requests from multiple users into one connection c, as in [16]. In such cases, b_c, p_c^{node}, and $\{p_{c,f}^{\text{func}}\}_{f \in \mathcal{R}_c}$ are multiplied by the number of aggregated users in the connection c, respectively.

B. Augmented Network

We consider the NFV network relies on a physical network $G = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ (resp. $\mathcal{E}$) is the set of physical nodes (resp. links), as shown in the middle layer of Fig. 1. G is a directed graph where an arrow from node $i \in \mathcal{V}$ to

$j \in \mathcal{V}$ expresses the physical link (i, j). For example, the bidirectional arrow between nodes v_1 and v_2 indicates that there are two physical links between them, i.e., (v_1, v_2) and (v_2, v_1), in Fig. 1. Each physical link $(i, j) \in \mathcal{E}$ (resp. each physical node $i \in \mathcal{V}$) has residual bandwidth $B_{i,j}$ (resp. residual processing capacity P_i) at the start of serving connections $c \in \mathcal{C}$. The NFV network totally supports a set of F functions, $\mathcal{F} = \{f_1, \ldots, f_F\}$. In general, there are two kinds of physical nodes: VNF-enabled nodes ($\mathcal{V}_{\mathrm{VNF}} \subseteq \mathcal{V}$) and normal nodes. Each VNF-enabled node $i \in \mathcal{V}_{\mathrm{VNF}}$ is capable of (part of) F functions, $\mathcal{F}_i \subseteq \mathcal{F}$, while the normal nodes are conventional routers and switches only for data forwarding. In what follows, for simplicity, we assume that all the nodes are VNF-enable (i.e., $\mathcal{V}_{\mathrm{VNF}} = \mathcal{V}$). From the viewpoint of each function $f \in \mathcal{F}$, f is assigned to N_f ($N_f > 0$) VNF-enabled nodes. In case of the service chaining, the location of each function f is fixed, and thus N_f is also constant. On the other hand, in case of both the service chaining and function placement, we aim to determine both the number N_f and locations of each function f according to the service chain requirements of connections $c \in \mathcal{C}$.

Service chaining for the connection c with the requirements $(o_c, d_c, \mathcal{R}_c, b_c, p_c^{\mathrm{node}}, \{p_{c,f_{c,k}}^{\mathrm{func}}\}_{k=1,\ldots,K_c})$ is finding an appropriate service path $\mathcal{S}_c$, which starts from o_c and ends with d_c while executing functions of $\mathcal{R}_c$ in the required order and passing through the corresponding physical nodes and links under the capacity constraints on both physical nodes and links. Inspired by the approach in [12], we can regard the service chaining as the SPTP. The SPTP tries to find a shortest path from an origin node to a destination node with the constraint that the path should visit at least one node from each of K ($K > 0$) given disjoint node subsets $\mathcal{T}_1, \ldots, \mathcal{T}_K$ [9]. In our case, $\mathcal{T}_k = \{\hat{v}_{f_{c,k}}\}$ ($k \in \mathcal{K}_c$), where $\hat{v}_{f_{c,k}}$ is an *imaginary node* responsible for kth function $f_{c,k}$ in $\mathcal{R}_c$. For example, in Fig. 1, $\mathcal{T}_1 = \{\hat{v}_{f_2}\}, \mathcal{T}_2 = \{\hat{v}_{f_3}\}$, and $\mathcal{T}_3 = \{\hat{v}_{f_1}\}$.

Note that imaginary nodes are not actual virtual nodes serving the corresponding functions but they play a key role in formulating the service chaining as the SPTP-based ILP. (The detail of formulation will be given in Section IV.) We further introduce two sets of virtual links, $\widehat{\mathcal{E}}^{\mathrm{in}}$ and $\widehat{\mathcal{E}}^{\mathrm{out}}$. $\widehat{\mathcal{E}}^{\mathrm{in}}$ is a set of links incoming to imaginary nodes, $\widehat{\mathcal{E}}^{\mathrm{in}} = \{(v, \hat{v}_f) \mid v \in \mathcal{V}, \hat{v}_f \in \widehat{\mathcal{V}}, f \in \mathcal{F}_v\}$. On the other hand, $\widehat{\mathcal{E}}^{\mathrm{out}}$ is a set of links outgoing from imaginary nodes, $\widehat{\mathcal{E}}^{\mathrm{out}} = \{(\hat{v}_f, v) \mid \hat{v} \in \widehat{\mathcal{V}}, v \in \mathcal{V}, f \in \mathcal{F}_v\}$. In this model, selecting an outgoing virtual link $(\hat{v}_f, v)$ from an imaginary node $\hat{v}_f$ to a physical node v as a part of the service path can represent executing the function f at the physical node v. For example, in Fig. 1, the virtual link $(\hat{v}_{f_2}, v_1)$ indicates that physical node v_1 is capable of function f_2. Note that physical node v_4 is also capable of function f_2 in this example. Selecting the virtual link $(\hat{v}_{f_2}, v_1)$ (resp. $(\hat{v}_{f_2}, v_4)$) means that function f_2 will be executed at physical node v_1 (resp. v_4).

We call the finally obtained network the *augmented network* $G^+ = (\mathcal{V}^+, \mathcal{E}^+)$ where $\mathcal{V}^+ = \mathcal{V} \cup \widehat{\mathcal{V}}$ and $\mathcal{E}^+ = \mathcal{E} \cup \widehat{\mathcal{E}}^{\mathrm{in}} \cup \widehat{\mathcal{E}}^{\mathrm{out}}$. An example of the augmented network is shown in the middle layer of Fig. 1. For simplicity in description, the neighbors of node i in G^+ is defined as $\mathcal{V}_i^+ \subseteq \mathcal{V}^+$.

At the last of this section, we clarify the difference between the augmented network and the existing network models: layered graph [14] and expanded network [13]. Fig. 2 illustrates the structure comparison among the three network models in case of the service chaining with $C = 1$. (The comparison in case of both the service chaining and function placement will be given in Section IV-B.) The layered graph, which is given in the right of Fig. 2, consists of $K_c + 1$ layers of the original physical network and multiple vertical links connecting two successive layers. A vertical link exists between the node $i \in \mathcal{V}$ at kth layer and that at $(k+1)$th layer ($k = 1, \ldots, K_c$) only when the node i is capable of the kth function $f_{c,k}$ of the chain request c. The expanded network is an extension of the layered graph by adding the pruning process. The pruning process excludes some links from the network by considering the fact that each physical link (resp. node) should have the bandwidth (resp. processing) capacity for at least one traversal of the connection c. Note that the actual number of times that a certain link/node is used in the service path may be more than one. In other words, the pruned network may still have physical links (nodes) without sufficient capacity for supporting the service path. However, the frequency of link/node usage cannot be identified until calculating the service path, which makes the problem more complex. We will see an example of multiple usage of the same link in the service path. (Please see Section III-C.)

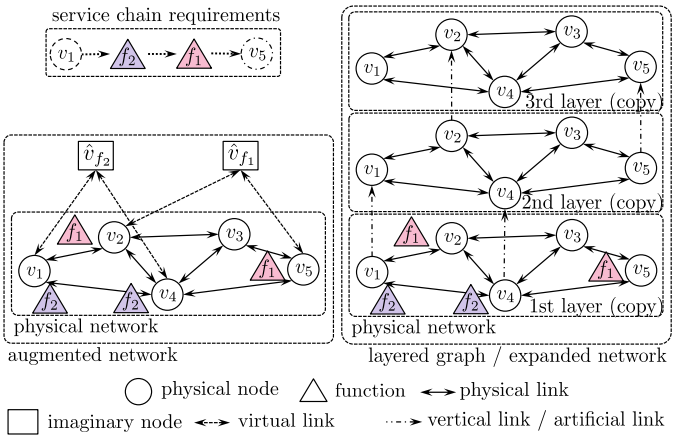


Fig. 2. Structure comparison among network models (service chaining case with $C = 1$).

TABLE II
SCALE COMPARISON AMONG NETWORK MODELS (SERVICE CHAINING CASE).

Network model	# of nodes	# of links
Layered graph	$\sum_{c \in \mathcal{C}} (K_c + 1)V$	$\sum_{c \in \mathcal{C}} ((K_c + 1)E + K_c N)$
Expanded net.	$\sum_{c \in \mathcal{C}} (K_c + 1)V$	$\sum_{c \in \mathcal{C}} ((K_c + 1)E + K_c N)$
Augmented net.	$V + \lvert \cup_{c \in \mathcal{C}} \mathcal{R}_c \rvert$	$E + 2 \lvert \cup_{c \in \mathcal{C}} \mathcal{R}_c \rvert N$

Table II presents the scale comparison among the network models in case of the service chaining. For simplicity, we assume that $N_f = N$ ($N > 0, f \in \mathcal{F}$). Note that the number of links in case of the expanded network is shown as its upper limit where all the nodes and links are available for the connection request. We can confirm that the augmented network becomes smaller than the layered graph and expanded

network with increase of K_c. Furthermore, all the networks are basically constructed per requested connections $\mathcal{C}$ (i.e., in an *on-demand* manner). In case of the augmented network, however, we can also construct a *full* augmented network that consists of physical nodes $\mathcal{V}$, physical links $\mathcal{E}$, and all F imaginary nodes with the corresponding $2N$ incoming/outgoing virtual links. Since the full augmented network can support any kind of connection request $\mathcal{C}$, we can save both the construction time and memory space of the network at the risk of increasing computation complexity.

C. Service Path

According to the SPTP, the service path $\mathcal{S}_c$ for $\mathcal{R}_c = (f_{c,1}, \ldots, f_{c,K_c})$ with the origin o_c and destination d_c can be expressed by a sequence of $K_c + 1$ subpaths, $(\mathcal{S}_{c,1}, \ldots, \mathcal{S}_{c,K_c+1})$, where the kth subpath $\mathcal{S}_{c,k}$ has the origin node $a_{c,k}$ and destination node $b_{c,k}$, which are given as follows:

$$(a_{c,k}, b_{c,k}) = \begin{cases} (o_c, \hat{v}_{f_{c,1}}), & k = 1, \\ (\hat{v}_{f_{c,k-1}}, \hat{v}_{f_{c,k}}), & k = 2, \ldots, K_c, \\ (\hat{v}_{f_{c,K_c}}, d_c), & k = K_c + 1. \end{cases}$$

$\mathcal{S}_{c,k}$ starts from its origin node $a_{c,k}$ and ends with its destination node $b_{c,k}$ through appropriate physical and virtual links in G^+. (Finding an optimal service path $\mathcal{S}_c^*$ is our goal and will be discussed in Section IV.) Note that there is no loop in each subpath but loop(s) may occur in the whole path, that is, some links may be used multiple times, which makes the service chaining problem more complex. For example, the bottom layer of Fig. 1 shows an example of the service path $\mathcal{S}_c = (\mathcal{S}_{c,1}, \ldots, \mathcal{S}_{c,4})$ where $\mathcal{S}_{c,1} = ((v_1, v_4), (v_4, \hat{v}_{f_2}))$ (red arrow), $\mathcal{S}_{c,2} = ((\hat{v}_{f_2}, v_4), (v_4, v_3), (v_3, \hat{v}_{f_3}))$ (orange arrow), $\mathcal{S}_{c,3} = ((\hat{v}_{f_3}, v_3), (v_3, v_2), (v_2, \hat{v}_{f_1}))$ (green arrow), and $\mathcal{S}_{c,4} = ((\hat{v}_{f_1}, v_2), (v_2, v_4), (v_4, v_3), (v_3, v_5))$ (blue arrow). We can also confirm the service path $\mathcal{S}_c$ in the augmented network, as shown in the middle layer of Fig.1. In this case, each subpath (i.e., $\mathcal{S}_{c,1}, \mathcal{S}_{c,2}, \mathcal{S}_{c,3}$, and $\mathcal{S}_{c,4}$) does not have any loop while the whole service path $\mathcal{S}_c$ has a loop. (The physical link (v_4, v_3) is used twice.)

We consider that the optimality of service path $\mathcal{S}_c$ is evaluated by total delay consisting of processing delay at nodes and propagation delay at physical links included in $\mathcal{S}_c$. (The detail will be given in Section IV.) Each physical link $(i, j) \in \mathcal{E}$ has the propagation delay $d_{i,j}^{\text{link}}$ ($d_{i,j}^{\text{link}} > 0$). As mentioned above, the connection c requires packet processing at each physical node that it uses, and thus the corresponding processing delay at the physical node $i \in \mathcal{V}$ is given by d_i^{node} ($d_i^{\text{node}} > 0$). In $\mathcal{S}_c$, each function $f \in \mathcal{R}_c$ is executed at the corresponding physical node v with the processing delay of $d_{\hat{v}_f,v}^{\text{func}}$ ($d_{\hat{v}_f,v}^{\text{func}} > 0$).

IV. Modeling Service Chaining and Function Placement as Shortest Path Tour Problem Based Integer Linear Program

In this section, we propose two kinds of SPTP-based ILPs: one is only for the service chaining, $\text{ILP}_{\text{SC}}^{\text{SPTP}}$, and another is for both the service chaining and function placement, $\text{ILP}_{\text{SCFP}}^{\text{SPTP}}$.

A. SPTP-based ILP for Service Chaining

Inspired by the ILP for the constrained SPTP in [12], we first formulate the SPTP-based ILP for the service chaining, $\text{ILP}_{\text{SC}}^{\text{SPTP}}$, which has the binary decision variables $x_{i,j}^{c,k}$ ($c \in \mathcal{C}, (i,j) \in \mathcal{E}^+, k \in \mathcal{K}_c^+$):

$$x_{i,j}^{c,k} = \begin{cases} 1, & \text{if physical/virtual link } (i,j) \text{ is used in } k\text{th} \\ & \text{subpath of service path for connection } c, \\ 0, & \text{otherwise.} \end{cases}$$

$$\min \sum_{c \in \mathcal{C}} \sum_{(i,j) \in \mathcal{E}^+} d_{i,j} \sum_{k \in \mathcal{K}_c^+} x_{i,j}^{c,k} \tag{1}$$

$$\text{s.t. } x_{i,j}^{c,k} = \{0,1\}, \qquad (i,j) \in \mathcal{E}^+, c \in \mathcal{C}, k \in \mathcal{K}_c^+, \tag{2}$$

$$\sum_{j \in \mathcal{V}_{a_{c,k}}^+} x_{a_{c,k},j}^{c,k} = 1, \qquad c \in \mathcal{C}, k \in \mathcal{K}_c^+, \tag{3}$$

$$\sum_{j \in \mathcal{V}_{b_{c,k}}^+} x_{j,b_{c,k}}^{c,k} = 1, \qquad c \in \mathcal{C}, k \in \mathcal{K}_c^+, \tag{4}$$

$$\sum_{j \in \mathcal{V}_i^+} x_{j,i}^{c,k} = \sum_{j \in \mathcal{V}_i^+} x_{i,j}^{c,k}, \quad i \in \mathcal{V} \setminus \{a_{c,k}, b_{c,k}\}, c \in \mathcal{C}, k \in \mathcal{K}_c^+, \tag{5}$$

$$x_{i,\hat{v}_{f_{c,k}}}^{c,k} = x_{\hat{v}_{f_{c,k}},i}^{c,k+1}, \quad (i, \hat{v}_{f_{c,k}}) \in \widehat{\mathcal{E}}^{\text{in}}, (\hat{v}_{f_{c,k}}, i) \in \widehat{\mathcal{E}}^{\text{out}}, c \in \mathcal{C}, k \in \mathcal{K}_c, \tag{6}$$

$$x_{i,\hat{v}_{f_{c,m}}}^{c,k} = 0, \quad (i, \hat{v}_{f_{c,m}}) \in \widehat{\mathcal{E}}^{\text{in}}, c \in \mathcal{C}, k \in \mathcal{K}_c^+, \hat{v}_{f_{c,m}} \neq b_{c,k}, \tag{7}$$

$$\sum_{c \in \mathcal{C}} \left(b_c \sum_{k \in \mathcal{K}_c^+} x_{i,j}^{c,k} \right) \leq B_{i,j}, \qquad (i,j) \in \mathcal{E}, \tag{8}$$

$$\sum_{c \in \mathcal{C}} \left(p_c^{\text{node}} \sum_{(v,j) \in \mathcal{E}} \sum_{k \in \mathcal{K}_c^+} x_{v,j}^{c,k} + \sum_{(\hat{v}_f, v) \in \widehat{\mathcal{E}}^{\text{out}}} p_{c,f}^{\text{func}} \sum_{k \in \mathcal{K}_c^+} x_{\hat{v}_f,v}^{c,k} \right) \leq P_v, \quad v \in \mathcal{V}. \tag{9}$$

The objective function (1) is the minimization of the total delay of all the service paths where $d_{i,j}$ is given as follows:

$$d_{i,j} = \begin{cases} d_i^{\text{node}} + d_{i,j}^{\text{link}}, & \text{if } (i,j) \in \mathcal{E}, \\ d_{i,j}^{\text{func}}, & \text{if } (i,j) \in \widehat{\mathcal{E}}^{\text{out}}, \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

We first observe that the objective function (1) is the same as that of the SPTP. From the viewpoint of service chaining, (10) indicates that passing each physical link $(i,j) \in \mathcal{E}$ suffers both forwarding delay d_i^{node} and propagation delay $d_{i,j}^{\text{link}}$. The service path also suffers the execution delay of each function $i \in \mathcal{R}_c$ at the corresponding physical node j. As a result, the objective function (1) can be rewritten by

$$\sum_{c \in \mathcal{C}} \left(\sum_{(i,j) \in \mathcal{E}} (d_i^{\text{node}} + d_{i,j}^{\text{link}}) \sum_{k \in \mathcal{K}_c^+} x_{i,j}^{c,k} + \sum_{(\hat{v}_f, v) \in \widehat{\mathcal{E}}^{\text{out}}} d_{\hat{v}_f,v}^{\text{func}} \sum_{k \in \mathcal{K}_c^+} x_{\hat{v}_f,v}^{c,k} \right),$$

where the first (resp. second) term corresponds to the physical forwarding and propagation delay (resp. function execution

delay). For example, in the bottom of Fig. 1, the horizontal (resp. vertical) arrows correspond to the physical (resp. virtual) links included in service path $\mathcal{S}_c$. Note that the objective function can be transformed into the minimum-cost flow problem by replacing $d_{i,j}$ with node/link utilization as in [13]:

$$d_{i,j} = \begin{cases} \dfrac{b_c}{B_{i,j}}, & \text{if } (i,j) \in \mathcal{E}, \\ \dfrac{p^{\text{func}}_{c,f(i)}}{P_i}, & \text{if } (i,j) \in \widehat{\mathcal{E}}^{\text{out}}, \\ 0, & \text{otherwise}, \end{cases} \tag{11}$$

where $f(i)$ represents the function for which the imaginary node i is responsible.

Constraints are given by (2)–(9). Constraint (2) represents the binary decision variables. Constraints (3)–(5) present the flow rules in each subpath $\mathcal{S}_{c,k}$ $(c, \in \mathcal{C}, k \in \mathcal{K}^+_c)$. Constraint (3) (resp. Constraint (4)) indicates that the origin (resp. destination) node $a_{c,k}$ (resp. $b_{c,k}$) of the connection c's subpath k has the outgoing (resp. incoming) flow. Constraint (5) is the flow conservation rule at each intermediate node i in the connection c's subpath k $(c, \in \mathcal{C}, k \in \mathcal{K}^+_c)$. For example, focusing on the 1st subpath in Fig. 1, we observe that the flow occurs at the physical node v_1 (i.e., $\sum_{j \in \mathcal{V}^+_{v_1}} x^{c,1}_{v_1,j} = 1$), goes through any physical node v (i.e,. $\sum_{j \in \mathcal{V}^+_v} x^{c,1}_{j,v} = \sum_{j \in \mathcal{V}^+_v} x^{c,1}_{v,j}$), and finally ends with the imaginary node $\hat{v}_{f_2}$ (i.e., $\sum_{j \in \mathcal{V}^+_{\hat{v}_{f_2}}} x^{c,1}_{j,\hat{v}_{f_2}} = 1$).

Constraint (6) guarantees the connectivity between two successive subpaths $\mathcal{S}_{c,k}$ and $\mathcal{S}_{c,k+1}$ of the connection c $(c, \in \mathcal{C}, k \in \mathcal{K}_c)$. More specifically, the connection c's $(k+1)$th subpath should start from the same physical node as the final physical node of the connection c's kth subpath. For example, focusing on the 1st and 2nd subpaths in Fig. 1, we observe that $x^{c,1}_{i,\hat{v}_{f_{c,1}}} = x^{c,2}_{\hat{v}_{f_{c,1}},i}$ $(\forall (i, \hat{v}_{f_{c,1}}) \in \widehat{\mathcal{E}}^{\text{in}}, \forall (\hat{v}_{f_{c,1}}, i) \in \widehat{\mathcal{E}}^{\text{out}})$ where $f_{c,1} = f_2$. Constraint (7) prohibits the imaginary node $\hat{v}_{f_{c,m}}$ from being used in the kth subpath $(m \neq k)$. For example, focusing on the 1st subpath in Fig. 1, we observe that $x^{c,1}_{i,\hat{v}_{f_{c,m}}} = 0$ $(m \neq 1, \forall (i, \hat{v}_{f_{c,m}}) \in \widehat{\mathcal{E}}^{\text{in}})$.

Constraint (8) gives the physical link capacity constraint where the total bandwidth consumption of the physical link $(i,j) \in \mathcal{E}$ should be equal or less than the residual bandwidth capacity $B_{i,j}$. The ratio of the left side to the right side of (8) is the utilization $u_{i,j}$ of the physical link (i,j). Similarly, constraint (9) shows the processing capacity constraint where the total processing load of the physical node $v \in \mathcal{V}$ should be equal or less than the processing capacity P_v. Note that the processing load consists of the traversal cost, $\sum_{c \in \mathcal{C}} (p^{\text{node}}_c \sum_{(v,j) \in \mathcal{E}} \sum_{k \in \mathcal{K}^+_c} x^{c,k}_{v,j})$, and the processing cost, $\sum_{c \in \mathcal{C}} (\sum_{(\hat{v}_f, v) \in \widehat{\mathcal{E}}^{\text{out}}} p^{\text{func}}_{c,f} \sum_{k \in \mathcal{K}^+_c} x^{c,k}_{\hat{v}_f, v})$. The ratio of the left side to the right side of (9) is the utilization u_v of the physical node v.

B. SPTP-based ILP for Service Chaining and Function Placement

The SPTP-based ILP for the service chaining problem $\text{ILP}^{\text{SPTP}}_{\text{SC}}$ in Section IV-A can easily be extended to an optimization problem, $\text{ILP}^{\text{SPTP}}_{\text{SCFP}}$, which considers not only the service chaining but also the function placement, in the following manner. In Section IV-A, the function locations are regulated by the augmented network where virtual links are only established between imaginary nodes and physical nodes that are capable of the corresponding functions. In other words, connecting each function (imaginary node) to all VNF-enabled physical nodes via virtual links, i.e., $\widehat{\mathcal{E}}^{\text{in}} = \{(v, \hat{v}_f) \mid v \in \mathcal{V}_{\text{VNF}}, \hat{v}_f \in \widehat{\mathcal{V}}, f \in \mathcal{F}\}, \widehat{\mathcal{E}}^{\text{out}} = \{(\hat{v}_f, v) \mid \hat{v} \in \widehat{\mathcal{V}}, v \in \mathcal{V}_{\text{VNF}}, f \in \mathcal{F}\}$, can consider all possibilities of function placement at the expense of the computation complexity. For simplicity, we consider $\mathcal{V}_{\text{VNF}} = \mathcal{V}$ in what follows.

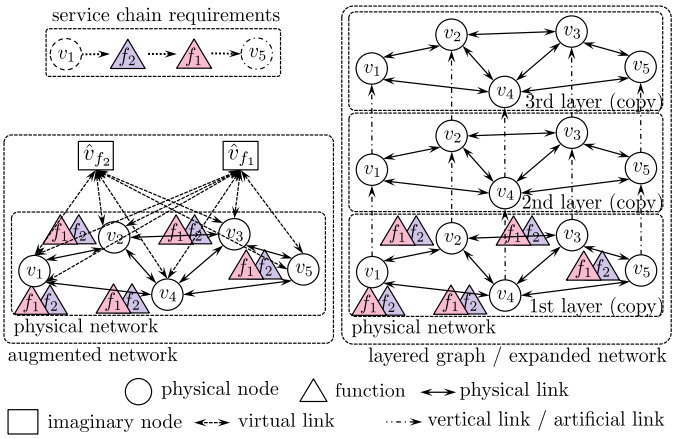


Fig. 3. Structure comparison among network models (service chaining and function placement case with $C = 1$).

TABLE III
SCALE COMPARISON AMONG NETWORK MODELS (SERVICE CHAINING AND FUNCTION PLACEMENT).

Network model	# of nodes	# of links
Layered graph	$\sum_{c \in \mathcal{C}} (K_c + 1)V$	$\sum_{c \in \mathcal{C}} ((K_c + 1)E + K_c N)$
Expanded net.	$\sum_{c \in \mathcal{C}} (K_c + 1)V$	$\sum_{c \in \mathcal{C}} ((K_c + 1)E + K_c N)$
Augmented net.	$V + \lvert \cup_{c \in \mathcal{C}} \mathcal{R}_c \rvert$	$E + 2 \lvert \cup_{c \in \mathcal{C}} \mathcal{R}_c \rvert N$

Fig. 3 illustrates an example of the augmented network for both the service chaining and function placement with $C = 1$, which is an expanded version of that only for the service chaining in Fig. 2. For comparison purpose, we also show the corresponding example of the layered graph and expanded network. Comparing Fig. 3 with Fig. 2, we can confirm that all the physical nodes are capable of all the VNFs (i.e., f_1 and f_2) and the corresponding virtual links (resp. vertical links/artificial links) exist in case of the augmented network (resp. layered graph and expanded network). Table III shows the corresponding number of nodes and links in case of the augmented network and expanded network. Comparing Table II with Table III, we observe that the number of nodes is identical while the number of virtual links (resp. artificial links) increases in case of the augmented network (resp. expanded network).

V. NUMERICAL RESULTS

In this section, we evaluate the effectiveness of the proposed SPTP-based ILP for the service chaining ($\text{ILP}^{\text{SPTP}}_{\text{SC}}$) and that for both the service chaining and function placement

($\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$). We solve the ILPs using the existing solver CPLEX 12.8 running on the server with Intel Xeon E7-8895v3 (18 cores and 2.60 GHz) and 2 TB memory.

A. Evaluation Scenario

We use the physical network consisting of 200 physical nodes and physical links. The physical links are randomly generated between two arbitrary physical nodes at the probability of $\pi = 0.032$ as in [32]. As for the physical node (resp. link) capacity, we assume that each physical node $i \in \mathcal{V}$ (resp. physical link between physical nodes $i \in \mathcal{V}$ and $j \in \mathcal{V} \setminus \{i\}$) has the same capacity $P_i = 1.71$ (resp. $B_{i,j} = 1.14$). We set the physical link delay between physical nodes i and j, $d_{i,j}^{\mathrm{link}}$, to be 10 [ms]. Each physical node v has the same traversal and processing delay, i.e., $d_v^{\mathrm{node}} = 1$ [ms] and $d_{v_f,v}^{\mathrm{func}} = 50$ [ms] ($v_f \in \mathcal{F}_v$), respectively.

In this paper, we mainly focus on (1) the scalability of $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ compared with the existing ILP over the expanded network ($\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{NGUYEN+}}$) [13] and (2) the performance improvement of $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$ compared with $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$. We consider a single-round resource allocation for online, batch, and offline processing. For each connection $c \in \mathcal{C}$, the origin node o_c and destination node d_c are randomly chosen from the physical nodes. Each connection c requires a set $\mathcal{R}_c$ of K_c functions, each of which is selected from the set of F functions. As mentioned in Section III-B, the number N_f of physical nodes capable of the function $f \in \mathcal{F}$ is fixed in case of the service chaining while that is adaptive in case of both the service chaining and function placement. The settings for $\mathcal{R}_c$, F, and N_f will be different in each scenario and explained later. We set the bandwidth requirement, processing requirement for traversing a node, and processing requirement of executing $f_{c,k} \in \mathcal{R}_c$ at a node as follows: $b_c = 0.1, p_c^{\mathrm{node}} = 0.05$, and $p_{c,f_{c,k}}^{\mathrm{func}} = 0.1$.

As for the scalability, we evaluate the computational complexity in terms of the execution time and deterministic time. The execution time is the actual time required to solve the problem. Note that CPLEX supports the parallel optimization and we set the number of threads to be 32. Since the execution time depends on the hardware spec of the server, we also use the deterministic time provided by CPLEX to evaluate the substantial complexity of the problem. CPLEX provides us with a choice between determinism and opportunism algorithms [33]. The determinism algorithm gives us a solution path in a deterministic manner while the opportunism one takes advantage of opportunities to improve performance. We adopt the determinism algorithm.

As for the service chaining problem, we compare $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ with $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{NGUYEN+}}$ proposed in [13]. Note that the authors proposed not only the ILP but also several heuristic algorithms to overcome the computation complexity in [13]. In addition, the objective function is the minimization of the link and node utilization to alleviate the blocking probability of connection requests. In what follows, focusing on the computation complexity of the ILP itself, we slightly replace the objective function of $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{NGUYEN+}}$ with that of $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$, i.e., the minimization of the total delay given by (1).

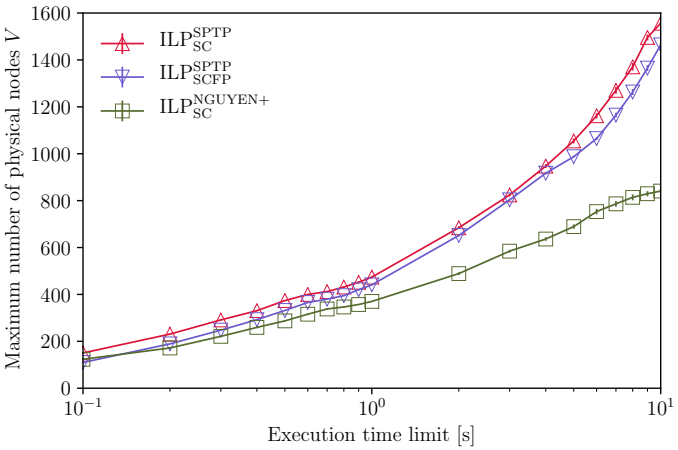


Fig. 4. The maximum number of physical nodes to be solved within the execution time limit.

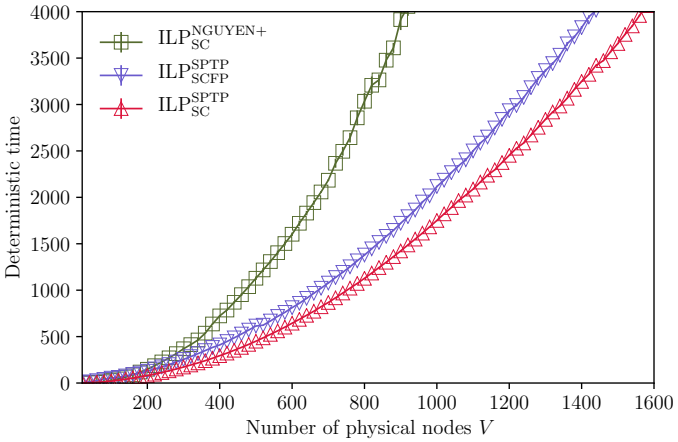


Fig. 5. Impact of the number of physical nodes on deterministic time.

As for the effectiveness of the service chaining and function placement, we evaluate the utilization of physical nodes/links, which are given in Section IV, as well as the total delay of all the service paths (i.e., objective value).

In what follows, we show the average of 100 independent numerical experiments.

B. Scalability

1) Scalability to Number of Physical Nodes: In this section, we examine the scalability to the number of physical nodes. The execution time may change even under the same number V of physical nodes, due to the difference of the topological structure. We focus on the maximum number of physical nodes that CPLEX can solve within a given time limit T and evaluate the average as follows. We first prepare 100 independent trials, each of which consists of problems with the different V ranging from $[20, 1500]$. For each trial, we solve each problem and obtain the execution time. For a given time limit T, we calculate the average of the maximum number of nodes, which could be solved within T, among the 100 trials. Note that the 95% confidence interval is also calculated.

Fig. 4 illustrates the maximum number of physical nodes that $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$, $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$, and $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{NGUYEN+}}$ can solve within

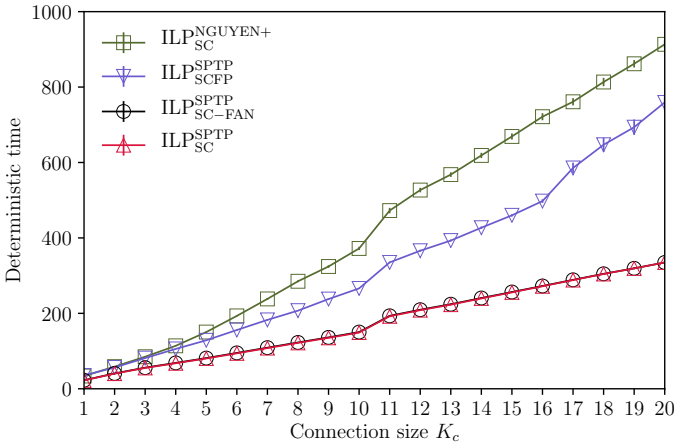


Fig. 6. Impact of connection size K_c on deterministic time.

the execution time limit where $C = 1, K_c = 5$, and $F = 20$. Note that K_c functions in $\mathcal{R}_c$ are randomly chosen from F functions such that $f_{c,k} \neq f_{c,m}$ $(k \neq m)$. In addition, $N_f = N = 5$ $(f \in \mathcal{F})$ for the service chaining problems. We first focus on the ILPs only for the service chaining, i.e., $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$ and $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{NGUYEN+}}$. We observe that $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ can solve 1.22–1.85 times as large-scale systems as $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{NGUYEN+}}$ under the same execution time limit. In particular, $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ can support 473 (resp. 1556) nodes within 1 (resp. 10) [s].

Fig. 5 presents how the deterministic time increases with the number of physical nodes, V, under the same settings. Note that we limit the deterministic time to 4,000 [ticks] because we find that the execution time of 10 [s] is almost equal to the deterministic time of 4,000 [ticks] under the experiment environments by comparing Figs. 4 and 5. We observe that the deterministic time of all the ILPs exponentially grows with increase of V, but the increasing rate of $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ is much smaller than that of $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{NGUYEN+}}$. As a result, $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ can solve the service chaining even in case of $V = 900$ while $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{NGUYEN+}}$ reaches the limitation of the deterministic time. As mentioned in Section III-B, the augmented network is more compact than the expanded network, which contributes to the scalability.

Next, we focus on the ILP for both the service chaining and function placement, i.e., $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$. Although $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$ has a more complex structure than $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$, we observe $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$ can support 441 (resp. 1464) nodes within 1 (resp. 10) [s] in Fig. 4. In addition, we also confirm that the deterministic time of $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$ increases by 9.9%, compared with $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ when $V = 1400$ in Fig. 5.

2) Scalability to Connection Size: Fig. 6 shows the impact of the connection size K_c on the deterministic time when $V = 200, C = 1$, and $F = 20$. Note that $N_f = N = 5$ $(f \in \mathcal{F})$ for the service chaining problems. In this evaluation, we set $\forall f \in \mathcal{R}_c$ to be different each other. As mentioned in Section III-B, the augmented network can be constructed as the full version. Therefore, we further show the results of $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ over the full augmented network ($\mathrm{ILP}_{\mathrm{SC-FAN}}^{\mathrm{SPTP}}$), in addition to those of $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$, $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$, and $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{NGUYEN+}}$. We first observe that the deterministic time almost linearly grows with increase of K_c, regardless of the ILP formulation. However, we also confirm that the deterministic time of $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ is always smaller than that of $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{NGUYEN+}}$. Specifically, $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ can reduce the deterministic time by 34.4% ($K_c = 1$) and 63.3% ($K_c = F$) compared with $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{NGUYEN+}}$.

TABLE IV
SERVICE CHAIN REQUIREMENTS [14], [16] (NAT: NETWORK ADDRESS TRANSLATOR, FW: FIREWALL, TM: TRAFFIC MONITOR, WOC: WAN OPTIMIZATION CONTROLLER, IDPS: INTRUSION DETECTION PREVENTION SYSTEM, AND VOC: VIDEO OPTIMIZATION CONTROLLER).

Service	Sequence of functions	Demand	b_c
Web service	NAT-FW-TM-WOC-IDPS	18.2%	100 Kbps
VoIP	NAT-FW-TM-FW-NAT	11.8%	64 Kbps
Video streaming	NAT-FW-TM-VOC-IDPS	69.9%	4 Mbps
Online gaming	NAT-FW-VOC-WOC-IDPS	0.1%	4 Mbps

Next, we also observe that the performance difference between $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ and $\mathrm{ILP}_{\mathrm{SC-FAN}}^{\mathrm{SPTP}}$ is limited, which indicates that $\mathrm{ILP}_{\mathrm{SC-FAN}}^{\mathrm{SPTP}}$ can deal with any kinds of connection requests with the pre-constructed full augmented network while suppressing the increase of the deterministic time.

Finally, we focus on the performance difference between $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ and $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$. Note that N_f $(f \in \mathcal{K}_c)$ is fixed to be $N = 5$ in case of $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ while that is adjusted according to the demand in case of $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$. Since C is set to be one in this evaluation, N_f $(f \in \mathcal{K}_c)$ also becomes one in case of $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$. We observe that the increasing rate of $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$ is larger than that of $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$. This result mainly stems from the different size of the augmented network between $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$ and $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$. The number of the imaginary links of $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$ (resp. $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$) increases by $2V$ (resp. $2N$) per connection size from Table III (resp. Table II).

3) Scalability to Number of Connections: In this section, we focus on the impact of the number of connections, C, on the computation complexity. We use the physical network with 200 nodes ($V = 200$) and set the capacity of each physical link (i, j) to be $B_{i,j} = 1$ [Gbps] $(i, j \in \mathcal{V}, i \neq j)$. As for the service demand, we use the more practical scenario in Table IV, which is given in [14], [16]. There are six function types ($F = 6$) and four service types, each of which consists of five functions ($K_c = 5$). For each connection $c \in \mathcal{C}$, we select one of the services according to the demand distribution. Every connection c serves ten aggregated users and the resulting processing requirements per connection for the functions are given in Table V. We also set p_c^{node} per connection to be 0.005. As for the physical node capacity, we assume that each physical node $i \in \mathcal{V}$ has the same capacity $P_i = 1$. Note that $N_f = N = 5$ $(f \in \mathcal{F})$ for the service chaining problems.

Fig. 7 depicts the impact of C on the deterministic time for $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$, $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$, and $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{NGUYEN+}}$. We first confirm that the deterministic time of $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ and $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$ is always smaller than that of $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{NGUYEN+}}$. Comparing the results of $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ and $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$, we observe that $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ initially has lower complexity than $\mathrm{ILP}_{\mathrm{SCFP}}^{\mathrm{SPTP}}$ but the relationship becomes reverted when $C \geq 17$. This is because $\mathrm{ILP}_{\mathrm{SC}}^{\mathrm{SPTP}}$ suffers scarcity of VNFs, due to the fixed value of $N_f = 5$

TABLE V
PROCESSING REQUIREMENTS PER CONNECTION (10 USERS) FOR THE VNFS.

Function type	$p^{\text{func}}_{c,f,k}$
NAT	0.0092
FW	0.009
TM	0.133
WOC	0.054
IDPS	0.107
VOC	0.054

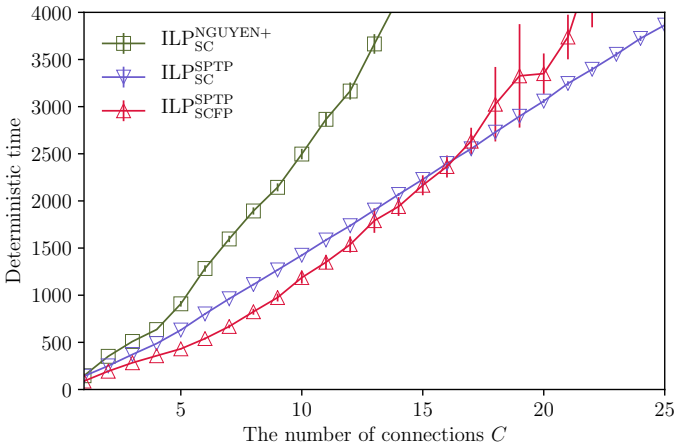


Fig. 7. Impact of the number of connections on the deterministic time.

($f \in \mathcal{K}_c, c \in \mathcal{C}$), while ILP$^{\text{SPTP}}_{\text{SCFP}}$ adjusts N_f according to its demand. Table VI presents the mean and standard deviation of N_f ($f \in \mathcal{F}$) in case of ILP$^{\text{SPTP}}_{\text{SCFP}}$ when $C = 20$. We confirm that all the functions except WOC require more than five VNFs. Since ILP$^{\text{SPTP}}_{\text{SCFP}}$ can appropriately determine not only N_f ($f \in \mathcal{F}$) but also the locations of functions, we will further examine the effectiveness of ILP$^{\text{SPTP}}_{\text{SCFP}}$ in Section V-C1.

4) Impact of Objective Function: Finally, we examine how the difference of the objective function affects the computation complexity. Figs. 8a, 8b, 8c, and 8d are the results in case of the objective function used in [13] (i.e., the minimization of node and link utilization), each of which corresponds to Figs. 4, 5, and 6, and 7, respectively. Note that the SPTP-based ILP can also support the minimization of node and link utilization by using $d_{i,j}$ in (11). We observe that the results in case of the minimization of node and link utilization are similar to those in case of the minimization of total delay.

C. Effectiveness of Service Chaining and Function Placement

Next, we focus on the effectiveness of the service chaining and function placement in terms of the allocated number of physical nodes capable of functions, the total delay, and the utilization of physical nodes and links. In this section, we use $V = 200$, the service chain requirements in Table IV, and the processing requirements in Table V, which results in $F = 6$ and $K_c = 5$.

1) Allocated Number of Physical Nodes Capable of Functions: As mentioned in Section IV, ILP$^{\text{SPTP}}_{\text{SC}}$ considers N_f ($f \in \mathcal{F}$) to be constant while ILP$^{\text{SPTP}}_{\text{SCFP}}$ adjusts both the number N_f and locations of physical nodes capable of functions.

TABLE VI
MEAN AND STANDARD DEVIATION OF N_f AMONG THE 100 INDEPENDENT NUMERICAL EXPERIMENTS ($C = 20$).

Function type	mean	std.
NAT	20.3	1.49
FW	19.92	1.22
TM	18.96	0.95
WOC	3.57	1.77
IDPS	16.69	1.55
VOC	13.46	2.04

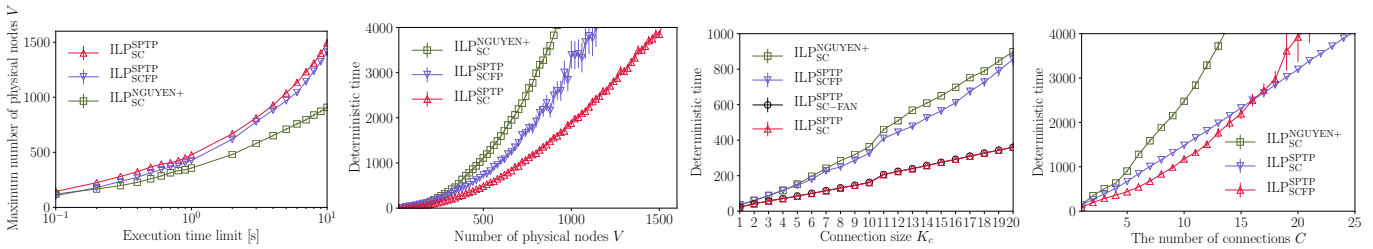
Fig. 9 illustrates how ILP$^{\text{SPTP}}_{\text{SCFP}}$ adjusts the number N_f of physical nodes capable of each function $f \in \mathcal{F}$ when the number C of connections changes. We show the demand for each function $f \in \mathcal{F}$, which is calculated by Table IV, in the legend and the corresponding supply (i.e., the percentage range of N_f) in the right side. We confirm that ILP$^{\text{SPTP}}_{\text{SCFP}}$ can appropriately adjust N_f for each function $f \in \mathcal{F}$ according to the corresponding demand.

2) Total Delay of All Service Paths: Next, we focus on the value of the objective function (1), i.e., total delay of all the service paths. Fig. 10 illustrates the impact of the number of connections, C, on the total delay of ILP$^{\text{SPTP}}_{\text{SCFP}}$ and ILP$^{\text{SPTP}}_{\text{SC}}$. To clarify the impact of locations of physical nodes capable of function $f \in \mathcal{F}$, we first obtain the optimal number N^*_f and locations of physical nodes capable of function f by solving ILP$^{\text{SPTP}}_{\text{SCFP}}$, and then we solve ILP$^{\text{SPTP}}_{\text{SC}}$ under the constraint of $N_f = N^*_f$. We observe that ILP$^{\text{SPTP}}_{\text{SCFP}}$ can reduce the total delay by 15.7%, compared with ILP$^{\text{SPTP}}_{\text{SC}}$ when $C = 20$. Since we consider homogeneous physical nodes and links in this evaluation (See Section V-A), this result indicates that the improvement of the total delay is mainly achieved by the decrease of the hop count of the service path. In particular, ILP$^{\text{SPTP}}_{\text{SCFP}}$ can reduce the average hop count of the service paths by 54.0%, compared with ILP$^{\text{SPTP}}_{\text{SC}}$ when $C = 20$.

3) Utilization of Physical Nodes and Links: Finally, we further examine the utilization of physical nodes and links. Fig. 11 (resp. Fig. 12) shows the average utilization of physical nodes (resp. links) for ILP$^{\text{SPTP}}_{\text{SCFP}}$ and ILP$^{\text{SPTP}}_{\text{SC}}$. Note that the setting of N_f for ILP$^{\text{SPTP}}_{\text{SC}}$ is the same as that in Section V-C2. We observe that ILP$^{\text{SPTP}}_{\text{SCFP}}$ can reduce the average utilization of physical nodes (resp. links) by 26.8% (resp. 60.9%), compared with ILP$^{\text{SPTP}}_{\text{SC}}$ when $C = 20$. Recall that both ILP$^{\text{SPTP}}_{\text{SCFP}}$ and ILP$^{\text{SPTP}}_{\text{SC}}$ support the same N_f for each function $f \in \mathcal{F}$ in this evaluation. In case of ILP$^{\text{SPTP}}_{\text{SCFP}}$, each function is appropriately assigned to physical nodes according to the sources and destinations of service requests, which realizes low latency, short hop, and low average node/link utilization.

VI. CONCLUSIONS

In this paper, we have formulated two kinds of the simple yet novel integer linear programs (ILPs) (i.e., ILP$^{\text{SPTP}}_{\text{SC}}$ for the service chaining and ILP$^{\text{SPTP}}_{\text{SCFP}}$ for both the service chaining and function placement) in network function virtualization (NFV) networks, with the help of the following two key ideas. One is focusing on the similarity between the service chaining problem and shortest path tour problem (SPTP) and another is the development of the new network model called



(a) The maximum number of physical nodes to be solved within the execution time limit.

(b) Impact of the number of physical nodes on deterministic time.

(c) Impact of connection size K_c on deterministic time.

(d) Impact of the number of connections on deterministic time.

Fig. 8. Impact of objective function on computation complexity (minimization of node and link utilization).

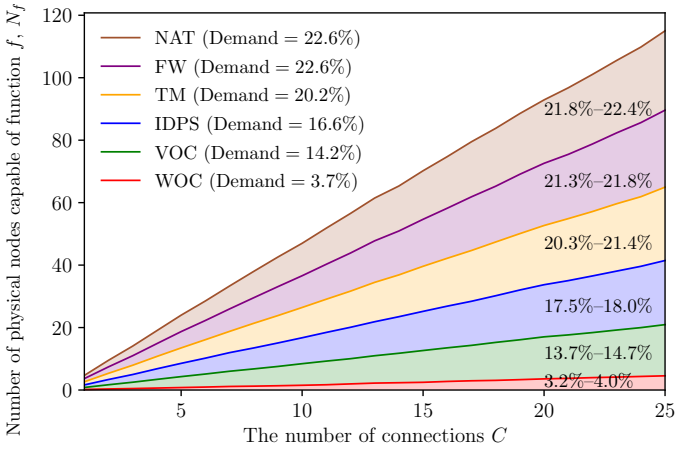


Fig. 9. Impact of the number of connections on the number of physical nodes capable of each function f.

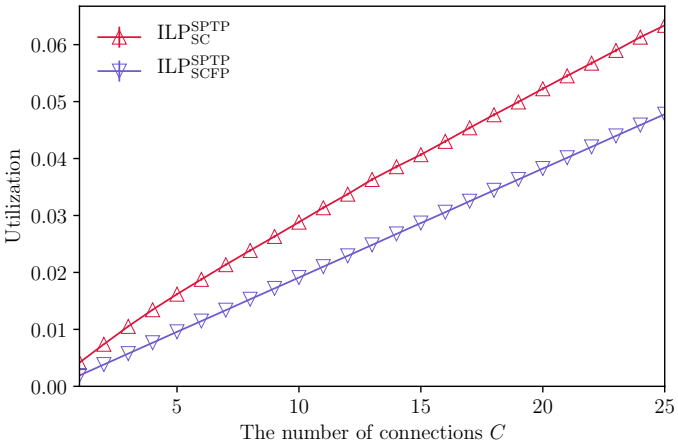


Fig. 11. The comparison of average utilization of physical nodes between ILP_{SCFP}^{SPTP} and ILP_{SC}^{SPTP}.

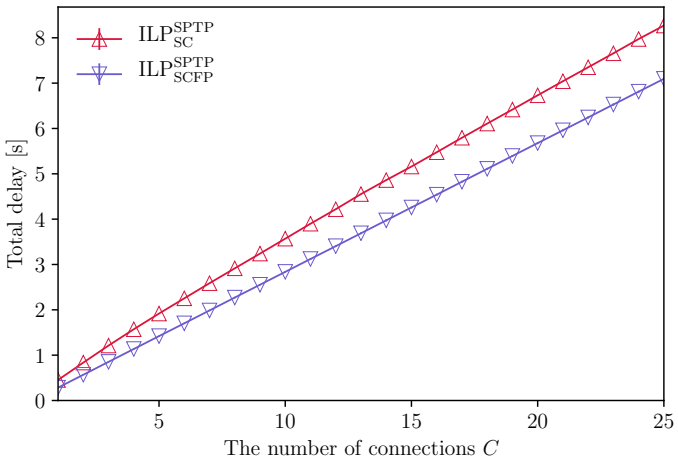


Fig. 10. Impact of the number of connections on the total delay.

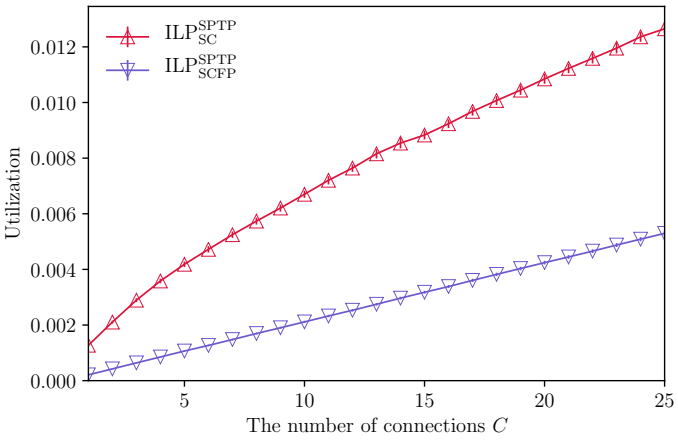


Fig. 12. The comparison of average utilization of physical links between ILP_{SCFP}^{SPTP} and ILP_{SC}^{SPTP}.

augmented network. Through numerical results obtained by solving them using the existing solver CPLEX, we have shown that ILP_{SC}^{SPTP} can support 1.22–1.85 times as large-scale systems as the existing ILP over the expanded network, $ILP_{SC}^{NGUYEN+}$. We have further demonstrated that ILP_{SCFP}^{SPTP} can reduce the total delay of all service paths by 15.7% and the average physical node (resp. link) utilization by 26.8% (resp. 60.9%) compared with ILP_{SC}^{SPTP}.

Since we have succeeded in revealing the exact relationship between the conventional SPTP and the NFV-related problems (i.e., service chaining and function placement), we expect that this work will open up a new vista of developing the SPTP-related heuristic algorithms to cope with larger scale NFV networks.

In this paper, we have mainly focused on revealing the fact that the NFV-related problems (i.e., service chaining and function placement) can exactly be connected to the conventional SPTP. We also plan the further extension of

the formulation to deal with more complicated NFV-related constraints (e.g., context switching costs and upscaling costs caused by multi-core processing and implementations [16]).

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