# Workshop on
# Spectral Geometry and Related Topics

**Date: October 30th (Fri) – November 1st (Sun), 2026**

**Place: Room 207, Building A, Faculty of Science, Nagoya University, Japan**

## Program

### October 30th (Fri)

• 13:30 – 14:30 Junya Takahashi (Tohoku University)

TITLE: Gap estimates for the eigenvalues of the Hodge-Laplacian and special Killing forms

ABSTRACT: On a closed Riemannian manifold $(M, g)$, we explore how the first eigenvalue $\lambda_1'^{(p)}(M, g)$ of the Hodge-Laplacian acting on exact $p$-forms depends on the degree $p$. If a closed Riemannian manifold admits a special differential form, such as a parallel form and a special Killing form, then we study the relationship between $\lambda_1'^{(p)}(M, g)$ and $\lambda_1'^{(s)}(M, g)$. As a consequence, we obtain an estimate for the gap $\lambda_1'^{(p)}(M, g) - \lambda_1'^{(s)}(M, g)$. This talk is based on joint works with Colette Anné (Nantes Universié in France).

• 15:00 – 16:00 Takumi Gomyou (Kogakuin University)

TITLE: Eigenvalue bounds for Graph Laplacians under Goldberg-Coxeter Constructions

ABSTRACT: The Goldberg-Coxeter construction is a subdivide operation for 3- and 4-valent graphs that produces a finer graph of the same valency. Recently, Omori, Naito, and Tate studied the spectral properties of graph Laplacians under the Goldberg-Coxeter construction and established general inequalities relating Laplacian eigenvalues before and after subdivision. However, the sharpness of these eigenvalue bounds has not yet been fully understood. We generalize their eigenvalue bounds by introducing a parameter function that reflects the subdivision structure. We then show that, for the tetrahedral graph, the resulting bound for the first Laplacian eigenvalue is sharp for every $(k, l)$-Goldberg-Coxeter construction.

• 16:30 – 17:30 Fan Kang (Yau Mathematical Sciences Center, Jingzhai, Tsinghua)

TITLE: Conformal Eigenvalues on Tori

ABSTRACT: In this talk, we discuss extremal problems for the first two nonzero eigenvalues of the Laplace-Beltrami operator on tori of unit area. We first review Berger's isoperimetric problem for the first eigenvalue and present a new proof using the conformal area method and Bryant's results on tori. We then turn to the second eigenvalue. Building on general bounds for the second eigenvalue on surfaces due to Karpukhin-Stern and Eddaoudi-Girouard, we obtain an improved upper bound within a fixed conformal class of the torus. This estimate provides further evidence for the Kao-Lai-Osting conjecture.

**October 31th (Sat)**

• 9:30 – 10:30 Akito Suzuki (Komatsu University)

TITLE: The True Extension of the Konno Distribution in 2D Quantum Walks

ABSTRACT: The weak limit theorem is foundational to quantum walk theory, which serves as the quantum counterpart to the classical central limit theorem. However, deriving exact two-dimensional limit distributions has been a major challenge for decades. By introducing a critical maximal speed parameter, we demonstrate that previous studies belong to a degenerate regime. Our work reveals two-dimensional Konno functions in the non-degenerate regime and completely describes the two-dimensional weak limit theorem.

• 11:00 – 12:00 Yuya Takeuchi (University of Tsukuba)

TITLE: CR Paneitz operator and embeddability

ABSTRACT: The CR Paneitz operator, a CR invariant fourth-order linear differential operator, plays a crucial role in three-dimensional CR geometry. It is deeply connected with global embeddability, the CR positive mass theorem, and the logarithmic singularity of the Szegö kernel. In this talk, I will present recent progress on the spectrum of the CR Paneitz operator, focusing in particular on how its nature differs between the embeddable and non-embeddable cases.

(break)

• 13:30 – 14:30 Keita Kunikawa (Tokushima University)

TITLE: Counterexamples to preservation of flat normal bundles under mean curvature flow

ABSTRACT: The study of submanifolds in higher codimension differs substantially from the hypersurface case because of the complexity of the normal bundle. The flatness of the normal bundle and the parallel principal normal condition are useful simplifying assumptions in the study of submanifolds. For example, minimal graphs in higher codimension are not necessarily stable in general, whereas flatness of the normal bundle implies stability. Despite the usefulness of these conditions, it has remained unclear for about twenty years whether they are preserved under mean curvature flow. In this talk, we construct explicit counterexamples showing that the flatness of the normal bundle is not preserved under mean curvature flow. We also give an example in which the normal bundle remains flat under mean curvature flow, while the parallel principal normal condition is not preserved.

• 15:00 – 16:00 Shin Kato (Osaka Central Advanced Mathematical Institute)

TITLE: Deformation of $n$-noids preserving flux

ABSTRACT: Let $X : M \to \mathbf{R}^3$ be a complete conformal minimal immersion of finite total curvature, and denote the nullity of $X$ by $\mathrm{Nul}(X)$. It is well known that $\mathrm{Nul}(X) \geq 3$ holds for any $X$, and that $\mathrm{Nul}(X) > 3$ holds if and only if its Gauss map $g$ is realized also as the Gauss map of some flat-ended non-branched or branched minimal surface (Ejiri-Kotani, Montiel-Ros). However, it seems to be difficult in general to clarify whether an arbitrarily given map $g$ satisfies the condition above or not. We want to consider this problem in the case that $X$ is an n-noid by means of its flux.

• 16:30 – 17:30 Yoshiaki Suzuki (Osaka University)

TITLE: Joint spectrum of the sub-Laplacian and the Reeb vector field on three-dimensional Heisenberg Bieberbach manifolds

ABSTRACT: A Heisenberg Bieberbach manifold is a compact quotient $\Gamma\backslash\mathbb{H}$ of the Heisenberg group $\mathbb{H}$, where $\Gamma$, called a Heisenberg Bieberbach group, is a discrete torsion-free subgroup of pseudo-Hermitian automorphism group of $\mathbb{H}$. In this talk, we classify three-dimensional Heisenberg Bieberbach groups up to conjugation by CR automorphism group of $\mathbb{H}$, and explicitly compute the multiplicities of the joint spectrum of the sub-Laplacian and the Reeb vector field on Heisenberg Bieberbach manifolds. This talk is based on joint work with Yuya Takeuchi (University of Tsukuba).

**November 1st (Sun)**

• 9:30 – 10:30 Sho Kubota (Aichi University of Education)

TITLE: Grover walks on finite graphs and their periodicity

ABSTRACT: The speaker is interested in the extent to which the eigenvalues of matrices associated with a finite graph determine its structure. In this talk, we discuss the problem of determining the structure of a graph through the periodicity of the Grover walk, which is a standard model of quantum walks on graphs. More specifically, we consider the following problem. Let $U(G)$ denote the time evolution matrix of the Grover walk on a finite graph $G$. For a given positive integer $\tau$, determine all graphs $G$ satisfying $U(G)^\tau = I$. Yoshie solved this problem for $\tau = 2, 4$ and for all odd $\tau$. In this talk, we present a recently obtained classification for the case $\tau = 6$. The proof uses the relationship between the spectrum of $U(G)$ and that of the normalized adjacency matrix (or equivalently, the normalized Laplacian matrix), and reduces the problem to characterizing graphs with certain eigenvalues.

• 11:00 – 12:00 Kei Funano (Tohoku University)

TITLE: Coarse spectral geometry of convex domains

ABSTRACT: In this talk I will explain a coarse version of the Kac problem.