Title: The Geometry of Extreme Value Distributions: Part 1

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Abstract: We provide a major new simplification and unification of the landmark results of univariate Extreme Value Theory. The ‘ Three Types Theorem ’ of Fisher and Tippett, Gnedenko ’s necessary and sufficient conditions for a distribution to be in the Domain of Attraction of one of the Fisher and Tippett types and Picklands ’ connection of this work with ‘ Peaks over threshold ’ results and Generalised Pareto distributions all are immediate consequences of the solution of an equivalence problem for distributions with respect to affine transformations.

We produce a new differential invariant which is the basis for our proofs of the classical Extreme Value Theory results of Fisher and Tippett, Gnedenko and Picklands. This invariant determines an intrinsic measure of the distance between location-scale equivalence classes of distributions. In particular it can be used to evaluate the rate of convergence of a distribution to its Extreme Value attractor.

We give new necessary and sufficient conditions for a distribution to be in the domain of attraction of one of the Three Types and for a distribution to have a Peaks over Threshold limit. In both cases the condition is the limiting value of our invariant which is simple to calculate and convenient to use in practice. We also show that there is a fundamental geometric relationship between the Three Types of Extreme Value Distributions and the Generalized Pareto Distributions.