Title: On geometry and symmetries of nonholonomoic distributions and curves of flags

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Abstract: I will discuss the following two problems in local differential geometry and the interplay between them: the equivalence of vector distributions (w.r.t. the action of the group of diffeomorphisms) and the equivalence of curves of special flags in a linear space w.r.t. the action of a subgroup G of the General Linear Group with fixed grading on its Lie algebra g. For the first problem the procedure of construction of canonical frame (coframe) was described by Tanaka in terms of the algebraic prolongation of the symbol of a distribution. The second problem can be treated in a similar way via the notion of a symbol of a curve of flags (which is a degree -1 element of g) and an appropriate notion of its algebraic prolongation in g (as a generalization of Se-ashi works).

The main point of the talk is that the first problem can be reduced in essence to the second one (with the Group G being the Linear Symplectic Group and flags being flags of isotropic/coisotropic subspaces) via so-called symplectification procedure taking its origin in Optimal Control Theory. In this way we introduce the notion of the Jacobi symbol of a distribution, which is a degree -1 element of a graded symplectic algebra and we describe the procedure of construction of canonical frame (coframe) for all distributions with the given Jacobi symbol in terms of natural algebraic operations with the Jacobi symbol in the category of graded Lie algebras. The main advantages of using Jacobi symbols of distributions compared to the classical Tanaka symbols are that the set of Jacobi symbols is discrete and all Jacobi symbols can be easily classified. Besides, the Jacobi symbol of a distribution is much coarser characteristic than its Tanaka symbols: distributions with different Tanaka symbols and even with different small growth vectors may have the same Jacobi symbol. The talk is based on the joint works with Boris Doubrov.