The lightlike geometry of spaclike submanifolds in Minkowski space

Shyuichi IZUMIYA
Department of Mathematics, Hokkaido University, Sspporo, JAPAN

Abstract

In [3] we have investigated an extrinsic differential geometry on spacelike submanifolds of codimension two in Lorentz-Minkowski space. Examples of codimension two spacelike submanifolds are given by hypersurfaces in Euclidean space, Hyperbolic space, spacelike hypersurfaces in Lorentz-Minkowski space and de Sitter space.

Recently, we have discovered a new geometry on submanifolds in Hyperbolic space which is now called the *horospherical geometry* in Hyperbolic space[2, 4]. The horospherical geometry is quite different from the hyperbolic geometry in Hyperbolic space. However, it has similar properties to the euclidean differntial geometry[2]. For example, the Gauss-Bonnet type theorem holds for the horospherical Gauss-Kronecker curvature.Moreover, a part of geometric meanings for horo-tightness introduced in [1] have been recently clarified in the framework of the horospherical geometry[5, 6].

The lightlike geometry for spacelike submanifolds in Lorentz-Minkowski space not only unifies the euclidean Differential Geometry and the horospherical geometry in Hyperbolic space but also implies the theory of spacelike hypersurfaces in Lorentz-Minowski space and de Sitter space. In this talk, we start to explain the previous results on the lightlike geometry of codimension two spacelike submanifolds in Lorentz-Miniowski space and give the framework for general spacelike submanifolds. One of the results is the Chern-Lashof type theorem for the lightlike total absolute curvature of spacelike submanifolds. Such a theorem naturally induces the notion of *lightlike convexity* and *lightlike tightness* etc.

References

[1] T. E. Cecil and P. J. Ryan, *Tight ant taut immersions into hyperbolic space*, J. London Math. Soc., **19** (1979), 561-572.

[2] S. Izumiya, and M. D. R. Fuster, *The horospherical Gauss-Bonnet type theorem in hyperbolic space*, Journal of Mathematical Scociety of Japan 58, 965–984 (2006)

[3] S. Izumiya and M. C. Romero Fuster, *The lightlike flat geometry on spacelike submanifolds of codimension two in Minkowski space*, Selecta Mathematica (NS) 13, 23–55 (2007)

[4] S. Izumiya, *Horospherical geometry in the hyperbolic space*, Advanced Studies in Pure Mathematics 55, 31-49 (2009)

[5] M. Buosi, S Izumiya and M. A. Soares Ruas, *Total absolute horospherical curvature of submanifolds in hyperbolic space*, Advances in Geometry 10, 603–620 (2010)

[6] M. Buosi, S Izumiya and M. A. Soares Ruas, *Horo-tight spheres in Hyperbolic space*, to appear in Geometriae Dedicata (2011)